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DEVOTED TO

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Journal
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Devoted to Science and the Mechanic Arts

Vol. 203 JANUARY, 1927 No. 1

LIGHT AND HEALTH.*

BY

CHARLES H. MAYO, M.D.

Rochester, Minnesota.

I FEEL it an honor to have the opportunity to speak in The Franklin Institute, dedicated to the memory of one of the world's most distinguished citizens. Franklin was a truly scientific man, full of the enthusiasm of research, and he yet found the time to be one of the four fire chiefs of the city of Philadelphia, furnishing his six leather water buckets and two cloth bags for salvage from burning houses.

The more one investigates cause and effect in the universe and the development of the world, the more respect one has for those ancient peoples of Central America, Egypt, and other countries who worshipped the sun. We pass over their sacrificial efforts to appease the wrath of the sun and win his favor since they are in harmony with many religious cults, in which the sufferers are sustained by the glory of martyrdom. Modern wars of civilized man are more destructive without doubt; those who are the greatest leaders in destruction become heroes. Concerning our universe there is little doubt that Chamberlain's theory is most acceptable, and that the sun's planetary system (of which we are a part) came from a most ancient near-accident to old Sol when another star body passed near and encircled it, without actually touching it, and caused the immediate development of such great heat that a number of masses were thrown off from it. The

* Presented at a meeting held Thursday, January 28, 1926.

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planets and satellites of these gaseous masses, made round by rotation and gravity, have since been cooling, each according to its size, as an enormous period of time has passed since that occasion. Mars probably has now about the climate of New York City. The period of the earth in its orbit is one year, while Neptune, so great is its orbit, has made the trip about the sun but eleven times since the birth of Christ.

Physicists, geologists and biologists have discussed the millions of years of time which have been required for the formation of the present stage of evolution and development much as if such eons were but minutes of the present. Sunlight has taken a comparatively short time to reach us, coming at the rate of 186,000 miles a second. It takes eight minutes and twenty seconds to make the trip of ninety-three million miles from the sun to the earth. Our sun is one of the smaller suns, that is, the stars emitting light, being more condensed than some which are larger. The earth, in comparison with the sun, would be but a grain of sand at twenty-three feet from it when the sun is represented by a tennis ball; the next nearest sun, another tennis ball, would be 1100 miles away. Only three suns other than our own are within ten light years of the earth, and from the nearest of them it would take four and a half years for the light to come to us. The space separating our solar system from the other suns is very great, considering the difference in time for their light rays to reach us compared with those from our sun. In fact, all life has been and is dependent on the sun as it goes on through the ages emitting radiant energy. Professor Russell says the sun will give satisfactory rays for another billion years, which relieves some of our worry. The heat rays from the sun are capable of being measured. It is giving off energy equivalent to the burning of billions of tons of coal each second, and we are getting only a small part of it. The heat received from the sun by the entire surface of the earth is equivalent to the burning of two million tons of coal every second. The heat from one hundred million of the stars (or suns) would not here raise the water in a thimble one degree in one hundred years. We have a universe, the bodies in which are slowly contracting, yet our earth that some think of as large for practical purposes is as much smaller as the rapidity of travel has increased in fifty years; a few decades ago it took weeks to sail to Europe but now it takes only a few days. On

land we had the horse, then came the rails, and now the automobile. The earth rolls on at the same old speed, and in just the degree that man's speed has developed in the last fifty, or one hundred, years actually to encompass it, just so much smaller may we consider our world and so much more interesting is it as we can visualize it in thought. Man has gradually come into control of the world's forces, principally by making use of the stored energy of the sun and its ever-present activity. The stored energy of the sun is in the form of coal which developed when a very different atmosphere existed, heavily charged with carbon, and giant rushes and masses of tropical growth, trees, ferns and grass, made enormous beds of deposits which, under pressure of layers of earth and rock, ultimately formed coal of various degrees of hardness. Lignite is half-formed coal produced when that age was passing, and peat, found in our northern marshes, the still less formed carbonaceous material of the present period. Such material, including wood, is capable of disorganization by fire, as it combines with oxygen. It thus gives up that heat in a short time which it was a long time in acquiring from the sun, gives back its carbon to the air and its minerals as a residue of ash. Water-power, or white coal, represents the rapid action of the sun. The energy of the sun stored in water as it is lifted by evaporation is released in the falling rain and the flowing river and can be transformed at man's desire and under his control.

In America we have made the greatest use of the sun's energy, and man's industrial labor is now backed by over sixty horsepower. The marvellous imagination of Jules Verne vividly recorded in his novels has, in fact, been surpassed by train and automobile, by airship, and both on and beneath the sea. However, the smaller our world, the greater is the interest taken in it and all that there is of life of every sort on earth. Our enjoyment of it comes from the sun.

Since the electronic theory was formulated, scientists have determined that our world is composed of thousands of combinations, 250,000 being known, of the ninety-two elements, with the first or lightest as hydrogen and the heaviest, uranium, number ninety-two. Of these elements ninety are now known and each of these elements drops into a pigeon-hole of numbered negative electrons or planets, so to speak, which compose its atom as a universe, and with positive electrons equal in number to its atomic

weight, which as protons make up its nucleus or sun. As Rutherford says, the size of the atom is equal in extent to the maximal-sized orbit of its negative electrons. It was Franklin who divided electric charges into positive and negative. To name them now they would be reversed. The fewer the movable electrons the more stable the material. The structure of the two missing elements is known, namely, those numbered 85 and 87, but the actual substances are still elusive. Only ninety-two electrons may be held together and those of high numbers are slowly breaking up of themselves; thus uranium breaks up into lead and radium, which is one hundred times more active, or again radium disintegrates into other radio-active forms, A, B, and C, and helium and lead; others can be changed by knocking an atom of hydrogen, the master element, out of them with powerful X-ray radiant energy. The iron core of the earth's centre approximates 4000 miles in diameter and is thus eight times larger than the moon. It is true that by absorption we gradually accumulate more from the radiant energy of the sun's heat waves alone than we lose.

All material has its specific atomic structure, even its definite electronic chemical combinations with other elements. Living material has its own electronic combination and activity of structure which is not necessarily the same as the ordinary combination of such elements but becomes so after death, with loss of radiant energy. Life, then, is more or less electric, and varyingly active, oxidative and chemical. Almost perfect acting cells are made chemically and resemble the living in some activities, especially in causing crystalline formation. Bacteria, ultra-microscopic, or larger and of molecular dimensions, may act as electrons of varying energy. Few, possibly 5 per cent., of the great mass of bacteria, which are the active living chemists of the world in nature, are destructive to living tissue. The most common forms of life in the sea are the bacteria engaged in forming limestone from the calcium in sea water. They work in sunlight and warmth now as in the Cambrian period of the world's history; in the strata laid down at that time the fossilized remains of the pre-Cambrian and Cambrian invertebrate life are held for our inspection.

It is but lately that we have appreciated radiant energy although it was only about two hundred and fifty years ago that

the sunlight was first broken by Sir Isaac Newton's prisms into its bands of light from violet to red. The visible rays represent but a fraction of the spectrum. Now we know that the heat waves lie in the red and infra-red regions and next come the radio waves which are many metres long and then the alternating electric current with some waves many kilometres long. At the other end of the visible spectrum is violet, and beyond the visible violet comes ultra-violet, then radium rays and X-ray with still shorter waves. The waves of the visible spectrum are measured by the millionth of millimetre length and run from about 400 to 800 millimicrons (violet to red, through the blue, green and yellow), or by the Ångström units (a ten-millionth of a millimetre) from approximately 4000 to 8000. Certain wave-lengths of ultra-violet rays are most important in stimulating the chlorophyl (which is the green of plant life), the hemoglobin of blood-cells (which, in thin layers, are also green), and the photosensitive plate. The ultra-violet is the most stimulating and is held by the tissues of the skin while shorter and longer waves at both ends of radiant energy pass through or are absorbed by the body. Thus red glass holds back all but the red waves of the light or visible spectrum and passes a considerable quantity of heat waves. Ultra-violet causes the cells of the skin to protect their nuclei rapidly by screening with melanin, or the so-called tan of sunburn. Such rays lower blood-pressure from 7 to 10 per cent., somewhat increase the oxygen of the blood and blood calcium, the activity of endocrine glands, and the storage of iodine by the thyroid. This is of great importance as the blood carries the same fourteen primary elements that good soil does for plant life. The ultra-violet increases vitamin A; in fact can develop it in linseed oil exposed to the ray. Cod-liver oil has a large amount of this vitamin. Thus the violet ray of the sun prevents and cures rickets, which is so prevalent among the children of Scotland with its fogs and clouds and smoky air, as it is approximately only for one-half year that they have much chance with old Doctor Sunshine.

Fortunately man's ingenuity has developed the quartz glass (or fused quartz) which permits the ultra-violet ray to pass. Celluloid and paraffined gauze are also somewhat permeable to it while common window-glass cuts out most of it. Thus the mercury-vapor quartz-lamp, or arc light with carbons combined with nickel, emits a large amount of ultra-violet which can be used in the

treatment of chronic diseases, especially tuberculosis of the lungs and joints. The greatest effect of ultra-violet from sunlight is obtained at midday as the rays pass through the thinnest layer of air over the earth. The long slanting rays of morning and afternoon are largely screened by the air, especially because of the average half-inch layer of water diffused in hydroscopic form throughout the air. Thus high mountain altitudes are used in order that such sun treatments shall be most effective, although ultra-violet treatments are of value for shorter periods in any place, and artificial ultra-violet light can be created where nature gives little or no aid with sunlight. The ultra-violet which can be transmitted through air covers one and one-half octaves of light radiation, and one of the most destructive bactericidal regions of ultra-violet light is just below the very limit of the solar spectrum, that is 2800 to 2900 Angströms. The ultra-violet stimulates chemical reactions without heat, which would otherwise require great heat to accomplish.

Water with carbon dioxide gas bubbling through it can be made into a hydrocarbon derivative formaldehyde, CH_2O, being formed by exposures to ultra-violet rays. Sugar is chemically but twelve parts of carbon with eleven of water. Sunlight acting on the chlorophyl of the leaf makes starch, $C_6H_{12}O_6$, while cellulose is formed by a loss of a molecule of water, as the leaf unloads its burden of starch to the tree trunk at night.

The first recorded use of light in medical treatment was at Margate in 1715, by Russell, who was stimulated to its use by watching the self-treatment of diseased animals. Finsen, of Copenhagen, gave it prominence in 1893, in the so-called Finsen ray; later Bernhard, in 1902, repeated the work by the employment of sunlight in the mountains of Switzerland for pulmonary tuberculosis; and in 1904, at Leysin in Switzerland, Rollier, finding that the treatment of tuberculosis of the lungs by heliotherapy under the direction of Bernhard was generally successful, started its use for the cure of so-called surgical tuberculosis, that is tuberculosis of bones, joints, glands, intestine, peritoneum and sinuses. The method is not a cure-all, but used with judgment and care may give surprisingly beneficial results. Those whose skins do not readily tan do not do as well as those whose skin quickly forms its melanin protection of the cell nucleus. In commencing treatment, the feet are exposed for a few minutes for a day or two, in

a few days the lower half of the legs, then later the legs to the knees, and ultimately the whole body. The time of day the skin is exposed makes a great difference as that light which gives the quickest photographic action has the greatest healing power.

The ultra-violet rays are destructive to skin if used without care. Probably this chemical reaction which is concerned in the destructive effects of the sun's rays on the skin in severe sunburn with blistering, destruction of skin and fever, consists of the formation of formaldehyde in the skin.

Hematoporphyrin made from blood and injected into the blood of the Caucassian makes him sensitive to light and may cause death in the sunlight, although without effect in the dark. Phyloporphyrin is its vegetable counterpart from certain foods like buckwheat; it causes local skin reactions in man and white-skinned animals when they are exposed to sunlight. When carefully used this produces no reaction, yet the effect of continued or repeated small doses is undoubtedly to develop resistance of the blood and tissue against some chronic and a few acute diseases.

The light of the ordinary incandescent bulb is due to a filament of carbon which is resistant to the passage of the current. The filament would burn up if oxygen were in the bulb, but it operates in a vacuum or a bulb partly filled with inert gas. Diathermy is a method of heating living tissues to a considerable depth by electric current (as in the case of the lamp) to a temperature much greater in the centre of a joint or muscle than can be obtained by superficial treatment. This temperature is greater than many of the germs of disease can withstand, and the process is similar to the pasteurizing of milk for thirty minutes at a temperature between 140° and 145° F. Such treatments are very effective in certain acute and chronic inflammations of muscles and joints, especially those caused by bacteria, as many bacteria are active in but a small range of temperature.

The essential elements when volatilized at high temperatures emit characteristic radiations. The spectrum of the sun's rays shows that the same elements which make up our earth are also present in the sun and this fact is one of the proofs of the earth's origin. Scientists were twenty-seven years studying a special band in the sun's spectrum, which they called "helium," before this element was finally found on earth. We now know that it can

be produced by the disintegration of radio-active substances, and is found both in the earth and in the sun.

Some recent writings on vision by Shastid call our attention to the development of vision. The amœba is usually selected as the example of a complete, active and industrious single cell. This cell opens at any point to engulf food and again opens at any point to pass out the waste. It moves by developing feet at will and changing shape, closing its projections and making new ones, the granules of its protoplasm serving within the cell as the endocrine glands of multi-cellular bodies. This cell is light-sensitive all over and is moved to action by it. The microbic *Euglena viridis* has a mouth near which is a red spot, sensitive to light and heat, the first eye. Later, eyes were placed in sockets, a condition found in some worms. These afterward filled with foreign material. As evolution advanced, the socket containing the sensitive spot was filled with a viscid substance, eye-fluid, which was often contaminated but which was later covered, in other forms of more advanced life, so that a cornea of a double convex form was developed. Later still came lids: These first appeared in fishes, but they were structurally incomplete.

Birds have the most wonderful eyes, which can be adjusted as telescopic eyes for great distances, with double sensitive retinal spots or foveæ. Thus the vulture and buzzard are called to the feast by sight, and not smell, which is but poor in them. In general it can be said that the sight of birds is far better than man's, being microscopic and telescopic. There are no blood-vessels in the retina; they see no blue and violet but see by the infra-red and red. In snakes the lids are transparent but are fused together. As the snake grows, this lid skin becomes cloudy and is cast off with the skin as the reptile grows and requires a new one. The snake has vertical inner eyelids. Since it is nearsighted, it sees objects only in motion; having poor vision at the best and being nearly deaf as well, it receives most outside impressions from the delicate forked tongue. Cats and owls control at will their pupils which are vertical for watching for birds up and down trees. Dodson says to those who love birds that the predatory cat makes good roses when buried beneath the rose bush. The horse has a transverse pupil; he can see behind to kick with but has little eye movement. The sun has burned out the lower part of his retina and

he sees little except below the eye level; in fact he often bumps his head when passing through doorways.

Undoubtedly eyes were first developed in sea life and evolved with many adaptations to environment. Some years ago I received several thousand small trout fry. These were placed in large tanks with running spring water and fed fine chopped liver. They were always quite hungry, although gradually much food accumulated in the bottom of the tank. When I stirred this up with a stick they took advantage of the opportunity to eat. Examining the trout of all sizes I found the lower lip a little longer than the upper; they could easily grasp what was over them but not under them. Their eyes were so placed as to see above and not below; a true game fish, living on insects or small fish and possessing sharp teeth. On the other hand, the sucker is an opposite type; the eyes are placed lower to see below, the upper lip longer and mouth turned down. Suckers were put in with the trout to keep the water clean, for they are scavengers without teeth, and also partly live on vegetable food. By examining fish from such a standpoint one can tell whether the flesh will be palatable as it is in trout or less so as in carp or suckers. Fishes with large eyes are found near the surface. In fishes a falciform ligament is attached to the back of the lens and to the back of the eye for focusing. They have no color vision. This fact will be disputed by those who sell artificial flies at high prices. When living confined to the dark water of caves fishes' eyes nearly or quite disappear and fishes at great depth in the sea, where little light penetrates, have small eyes and poor eyesight. Exposing such cave-fish to red rays will develop the eyes in one generation.

The housefly has both compound and single eyes, the compound ones being used for vision at a distance of some yards. Each eye-bulb has 4000 hexaform tubes, a nerve with pigment at the base in each tube and a double convex cornea over each. Between the compound eyes are three single eyes for use in near vision, at one or two inches.

It is certain that man's eyes were made for distant vision, chiefly required when he was an outdoor animal. Now he lives in the "grand canyons" of the city streets or passes his life indoors and as age comes on, or even before, he needs glasses for reading, and makes but little use of his distant vision. Samuel

Pepys in the seventeenth century gave up the personal writing of his interesting diary in his late thirties for lack of glasses. Many people are color blind and may see red only as a gray. Man with his need for near work and town life, reading and studying, is growing to use the eyes for the most part in near vision. He must have better accommodation from his lenses; the need should develop it. In city life he sadly misses the third eye in the back of his head, which is now but a vestigial structure, the pineal gland. Some species of life have a poorly formed third eye which disappears during the early period of life; the tuatara, a native lizard of New Zealand, has such a third pineal eye which persists throughout life and is the only one of this type that is found at the present time. The giant lipoid has a hole in the back of his skull for such an eye. Non-primates have no, or little, overlapping of the visual fields of the two eyes and no stereoscopic vision; many have a third eyelid, or membrane nictitans, like the horse, for cleansing the cornea of dust. All primates have a macula or good spot of vision. The vision of man is usually best in the right eye unless he is left-handed. The eyes of man are probably growing closer together; in the early embryonic life they are set wide apart, as in the horse. Shastid says that eventually both may rest in one socket; this may be a cause of argument in a half million years.

In a democracy the mass of the people are dependent on city officials for health protection. Seventy per cent. of the people of this country are now residents of the city and their needs should not be based on the working necessities of the 30 per cent. of farmers who are opposed to daylight saving. Those in the eastern portion more generally can be thankful to their officials who have put in daylight saving, making it possible for the children to receive more sunshine, and grown-ups a long evening and more contact with nature.

New Inorganic Complexes.—HARRY A. ALSENTZER, JR. (*Thesis*, Graduate School, University of Pennsylvania, 1926, 25 pages) has prepared normal tungstates and molybdates of chlorpentammine and brompentammine cobaltic complexes, as well as an ammonium cobaltimolybdate, and a green ammonium cobaltous tungstate. J. S. H.

AN ATTEMPT TO DETECT A CORPUSCULAR RADIATION OF COSMIC ORIGIN.

BY

W. F. G. SWANN, D.Sc.

Chairman of the Advisory Committee on Research of the Bartol Research Foundation of The Franklin Institute; Director of the Sloane Laboratory, Yale University; Member of the Institute.

BARTOL RESEARCH FOUNDATION. Communication No. 8.

CERTAIN theories of atmospheric electric phenomena have invoked the assumption of a corpuscular radiation of high penetrating power which has been supposed to enter our earth and participate in the maintenance of its charge which, in the absence of some replenishing action, would disappear on account of the vertical conduction current resulting from the operation of the potential gradient in our conducting atmosphere. In some cases the supposed corpuscular radiation has been attributed to extra terrestrial sources, while in others its origin has been sought in the atmosphere itself. It is natural to attempt a search for such a radiation by investigating the charging action which it might be supposed to have on an insulated body exposed to it. In 1915 I attempted such an experiment, using as the body on which the charging effect was sought a copper bar about 25 cm. long and 5 cm. in diameter. This copper bar could be inserted in a copper sheath which fitted it tightly and which was connected to an electrometer, the whole being shielded by a large copper can. The plan was to observe the rate at which the electrometer charged when the copper sheath was empty and when it was filled with the bar. The experiments gave no indication of any charging effect. I have always felt that it would have been desirable to have used a larger mass of metal, since a corpuscular radiation which had passed through the whole atmosphere, a layer of air comparable in absorbing power with a column of mercury 76 cm. high, without such absorption as would make itself strikingly evident, would hardly show appreciable absorption in a small thickness of metal. Moreover, the small diameter of the bar reduced the efficiency of the apparatus as regards catching any radiation which came in a direction appreciably different from the vertical. I had a larger piece of metal constructed for a more delicate test, but the change of my field of activities about that time prevented my pursuing the matter further.

In 1918, E. von Schweidler[1] published a very interesting account of experiments to detect a corpuscular radiation of cosmic origin. Von Schweidler used as the absorbing body a plate of iron of area 476 sq. cm., and thickness 2 cm. in some experiments and 4 cm. in others. This plate was connected to an electrometer, and was of course surrounded by a shield. Two methods of measurement were employed. In one, the absorbing plate was allowed to charge up until the charging current balanced the leak-

FIG. 1.

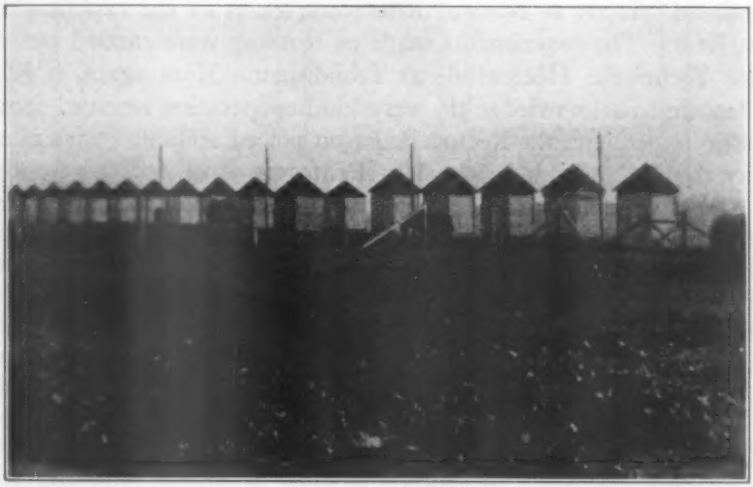
age due to the natural ionization in the surrounding vessel, while in the other method a potential was applied to the outer vessel so as to obtain the saturation current of the natural ionization. By measuring the saturation currents with potentials of opposite sign, any charging current of the type sought could be deduced as one-half of the difference of these saturation currents. As a result of his experiments, von Schweidler came to the conclusion that if there is a corpuscular current of sufficient amount to account for the maintenance of the earth's charge, less than 1 per cent. of it is absorbed by a plate of iron 4 cm. thick.

It is not unreasonable to suppose that the region of precipitation of a current of negative corpuscles might be determined to

[1] E. VON SCHWEIDLER, "Über Versuche sum Nachweis einer negativen Korpuskularstrahlung komischen Ursprungs," *Akad. Wiss. Wien.*, Berlin, **127**, pp. 515–533, 1918.

some extent by the earth's magnetic field, and that precipitation, if it occurs, might be more copious in those regions of the earth where the aurora is to be seen than in lower latitudes. In 1925 the writer devised a new form of apparatus for the purposes of such a test. The nature of the apparatus was such as to secure compensation against all disturbing influences normally inherent in electrostatic measurements, including ionization currents caused by the conductivity of the air in the apparatus. The absorbing

FIG. 2.



body was a cylinder of copper 20.6 cm. in diameter and 19.3 cm. high, and it was possible to work with an electrometer sensitivity of about 5000 millimetres per volt at a scale distance of one metre, *i.e.*, an electrometer sensitivity about twenty times that used by von Schweidler. The advantage of this was that readings could be obtained with the necessary accuracy in comparatively short times.

The apparatus was constructed under my direction at the Bartol Research Foundation of The Franklin Institute, which also financed the experiments and to whom I wish to express my sincere appreciation. The apparatus, which weighed altogether about 800 lbs., was shipped from Philadelphia to London, England. Through the courtesy of Prof. H. L. Callendar, I was able to set it up at the Imperial College of Science and Technology for the purpose of getting it into working shape, and it was then

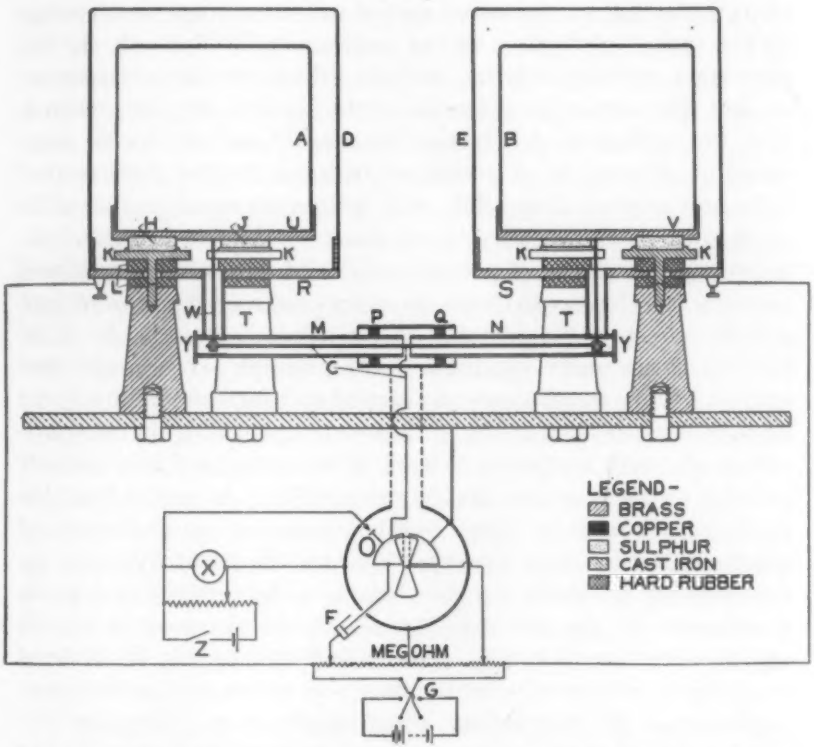
transported to Kew Observatory, where, through the kindness of the Superintendent of the Observatory, Dr. R. S. Whipple, I was able to make a set of measurements for the purpose of comparison with those made subsequently in Norway. I had planned, through the courtesy of Dr. G. C. Simpson, Director of the Meteorological Office, to make a set of observations at the Government Observatory at Lerwick, in the Shetland Islands, this observatory being in the auroral belt. As matters turned out, however, a delay in the arrival of the apparatus in England made it necessary to proceed directly to Norway after completion of the experiments at Kew. The experiments made in Norway were carried out at the Technische Höchschule at Trondhjem. Here again, it is a pleasure to acknowledge the very kind coöperation received from various gentlemen. Rector Pedersen placed suitable quarters at my disposal. I am indebted to Professor Anse for the trouble he took with the various arrangements which had to be made in connection with the arrival of the apparatus. I am indebted to Dr. O. Devik for his kindness in placing the facilities of the Physical Institute at my disposal, and to Mr. J. Wetlesen, who assisted me in the observations. To all of these gentlemen I am further indebted for the very courteous hospitality shown me personally. To the British Minister at Oslo I am indebted for his kind offices which facilitated the entry of the apparatus into Norway; and, finally, I have to thank the Norwegian customs authorities, who did everything in their power to minimize the technicalities of entry.

It was naturally desirable to carry out the experiment in a building which afforded as little obstruction as possible between the apparatus and the sky. Fig. 1 is a photograph of the building in which the experiments were carried out at Trondhjem. In it there was only one thin sloping roof to separate the apparatus from the sky. Indeed, the Höchschule was provided with almost humorously remarkable facilities for an experiment of this kind, for there exist there no less than seventeen small huts, Fig. 2, all provided with single roofs and capable of being kept at constant temperature. These had been built in connection with certain experiments on household heating and any one of them represented a structure which I might have considered myself happy to find for the purposes of the experiment. At Kew, I was also fortunate in finding a suitable single-roofed building.

APPARATUS AND METHOD.

Fig. 3 is a diagram of the apparatus used.[2] The general principles of its operation are as follows: *A* and *B* are two similar insulated hollow vessels of wall thickness about 1 mm. They were connected together by the wire *C*, and to one quadrant of an

FIG. 3.



electrometer. These vessels were each surrounded by two larger vessels *D* and *E*, of about the same wall thickness, which were connected to the opposite ends of a battery of about 250 volts, shunted by a megohm, the midpoint of the megohm being connected to the case of the electrometer. In this way equal and opposite potentials were applied to the vessels *D* and *E*. As a matter of fact, the potentials applied were sufficient to produce

[2] The upper portion of the drawing consists of a front elevation of the apparatus, the lower portion is diagrammatic, and represents the connections from the main apparatus to the electrometer and its appurtenances.

saturation currents, so that since, on account of the symmetry, the saturation currents corresponding to the two halves of the system should be equal and opposite, the rate of deflection of the electrometer should, theoretically, be zero. If, having measured the residual rate of deflection, we fill one of the vessels A or B with a solid mass of copper, and repeat the experiment, any charging of the solid mass of copper will show itself by a change in the rate of deflection of the electrometer. Not only do the saturation currents balance, but the effects on the electrometer of any fluctuations in potential of the battery are compensated. For, the coefficient of induction between D and the system composed of AB and the electrometer quadrant, and the coefficient of induction between E and that same system are equal, so that since on account of the equality in resistance of the two halves of the megohm, fluctuations of potential of D and E are always equal and opposite in reference to the electrometer case, such fluctuations produce no effect on the potential of the system composed of A, B, and the electrometer quadrant. The principle of compensation against battery fluctuations was carried still farther by the scheme of connections shown in Fig. 3. Instead of providing the electrometer needle with a separate battery, it was connected to a suitable position on the megohm already referred to. A small adjustable condenser F, with its inner member connected to the insulated quadrant and its outer member connected to a suitable point on the megohm, enabled a complete balance to be obtained as regards fluctuations of the potentials on D, E, the electrometer needle and the outer member of F. This balance could easily be obtained to a degree enormously higher than was necessary for the compensation of any fluctuations which might occur. The plan for obtaining the balance was to adjust matters so that on reversing the whole battery by means of the switch G, with the quadrant insulated, no deflection was obtained in the electrometer. It was readily possible to adjust conditions so that the residual deflections on the electrometer scale were only of the order of a millimetre at a sensitivity of 5000 millimetres per volt, so that variations of electrometer readings due to mere fluctuations of the potential of the battery were eliminated to a very high order of accuracy.

The following points in the design of the apparatus are worthy

of comment: The bases of the vessels *A, B, D, E,* were made of brass, and were formed as shown so that the remaining part of each vessel was in the form of an open copper can which could readily be placed in position open end downwards. The vessels *A* and *B* were each insulated by three sulphur cylinders, two of which are shown at *H* and *J*. Leakage from the outer vessels across these sulphur cylinders was prevented by means of metal guards *K* which were connected to the base of the whole instrument and insulated from the outer vessels by hard rubber *L*. The base of the apparatus was of course connected to the electrometer case. The metal guards extended out so that the hard rubber was well shielded from the vessels *A* and *B*. I have found this a matter of considerable importance in refined electrostatic measurements. A guard ring will shield the system to be protected from direct charging by the passage of electricity to it across the insulation, but charge motion, soakage, etc., in the insulator (in this case the hard rubber) which is subjected to the large potential difference will, by inductive action, cause considerable changes of potential of the main insulated system unless the guard extends out in such a way as to shield that system from the insulator across which there is a high potential. The improvements of conditions resulting from attention to this point are rather astonishing to one who encounters them for the first time. For similar reasons, since the tube *PQ* which connects to the electrometer has to be insulated from the shielding tubes *M* and *N* which are at the potentials of *D* and *E,* respectively, it is necessary to arrange the insulating hard rubber rings *P* and *Q* so that they are shielded from the central insulated rod *C*.

The guards *K* with their hard rubber supports *L* and the brass bases *R, S,* formed a unit with the brass pillars *T,* which were permanently attached to the iron base of the apparatus. The upper surfaces of the guards and the lower surfaces of *U* and *V* were recessed to receive the sulphur cylinders. These sulphur cylinders were not inserted until the apparatus was set up for an experiment, and then, previous to insertion, they were scraped with a piece of broken glass, after which they were passed through a flame to remove charge. After they were placed in position, the pieces *U* and *V* were fitted over them, and the two rods *W* were connected by the wire *C*. To facilitate this opera-

tion, the ends Y of the tubes M and N were provided with caps.

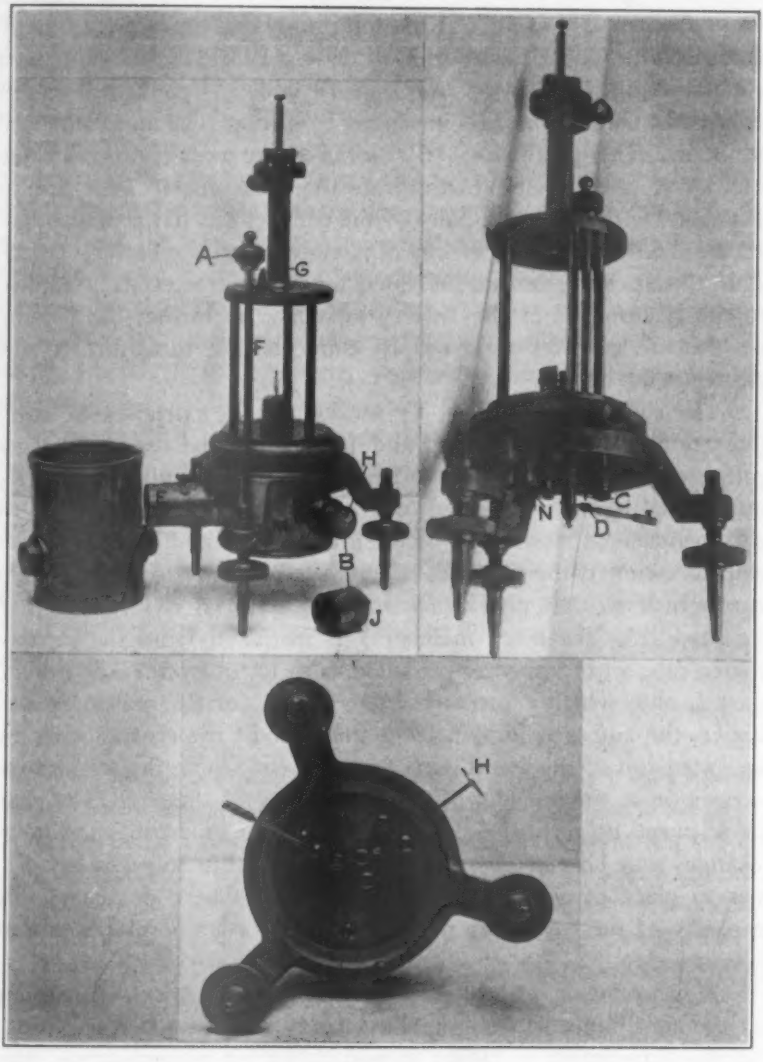
To facilitate the manipulation of the copper weight, which weighed about 57 kilograms, the chain hoist shown in Fig. 4 was used. This worked very smoothly, so that by its use there was no difficulty whatever in moving the weight from one place to another without unduly disturbing the apparatus, although, as a precaution, the compensation adjustment was made, and the electrometer

FIG. 4.

sensitivity was measured each time the disposition of the weight was changed. When it was desired to insert the solid copper cylinder, the copper cans were removed from the corresponding side, and the cylinder was brought into position and lowered so as to rest on the part U or V, as the case might be. The copper can A (or B) was then slid over it, and finally the outer can D (or E) was placed in position.

The electrometer used was of a type designed by the writer. It is shown, partially dissected, in Fig. 5, and the main respects in which it differs from the usual types are in the possession of an enclosed key A, which eliminates the necessity of using a separate shielding box, and the compensating condenser B. The key makes its contact between a silver wire C and a silver plate D

FIG. 5.



which is to be found underneath the electrometer and which is fastened to a rod coming straight out from the quadrant terminal through the shielding tube E which forms part of the metal housing M. The movable part of the key is, however, manipulated from the top. The weight at the top causes this movable part to go down solidly when it is allowed to do so. While the movable part slides up and down inside the tube F, which is connected to the case of the instrument, it is insulated from that tube and connected to the insulated terminal G. By means of the connections indicated in Fig. 3, it is possible to calibrate the electrometer with ease at any time, without altering any of the connections to the main apparatus. If the electrometer key O is closed, and the battery switch Z is closed, the potential indicated by the voltmeter X is applied to the quadrant. If the battery switch Z is raised, the closing of the electrometer key simply connects the quadrant to the electrometer case.

The condenser B (Fig. 5) serves for the purposes of compensation referred to above, and it may also be used for other purposes, such as the determination of the quantity sensitivity as distinct from the voltage sensitivity, or the determination of the coefficient of induction between the needle and quadrant. Its central member, the plate H, is connected directly to the quadrant through N so that no extra insulating material is introduced in its support. The outer member J is insulated from the electrometer case with amber K provided with an outer threaded metal part L onto which J screws. Here again, for the reasons stated above, the amber is hidden from the view of the central parts by an extension of the metal parts by means of which the attachment screws on to the housing M. The capacity of the condenser may be adjusted by screwing the outer member J in or out, and large changes may be made by replacing the central member by a cylinder in place of a disc. In the experiments here described, the capacity of the condenser was only of the order of magnitude of a centimetre.

Measurements of the Necessary Capacities, Etc.—In so far as the charging effect obtained was very small, so that the result of the experiment is rather to emphasize its insignificance than record its amount, a rather careful consideration of the methods concerned in measuring the capacities, etc., may seem out of place.

Nevertheless, since it is a fact that the theory of the use of a quadrant electrometer is frequently misunderstood, we shall deal with these matters fairly completely. That the potential on the needle has an effect on the quantity sensitivity is generally known, but the effect is frequently regarded simply as the equivalent of an increase of capacity of the system, a view which is justified so long as one realizes that the said increase of capacity is a function of the potential of the needle and the potential sensitivity of the electrometer. For given needle potentials it will vary, for example, with the thickness of the fibre suspension. For low sensitivity measurements these considerations play but little part in practice; but for sensitivities of the order of 5000 divisions per volt they are frequently by no means negligible. For these reasons, we shall deal with the matter of calculation of the quantities of electricity from the measured data with a degree of detail which would perhaps not be warranted for the special case in hand.

According to the well-known theory of the measurement of a quantity of electricity by means of an electrometer, if the central copper cylinders, together with their connections, including one pair of quadrants of the electrometer, are designated as the body (1) and if the needle of the electrometer is designated as the body (3), and if the q's refer to the coefficients of capacity and induction, the quantity of electricity Q on the body (1) in the undeflected position of the needle (denoted by the subscript zero) is

$$Q = (q_{11})_0 V_1 + (q_{13})_0 V_3 , \tag{1}$$

where V_1 is the potential of the system (1) and V_3 that of the needle; the potential of all other bodies (which are connected together) being taken as the arbitrary zero from which V_1 and V_3 are measured. If, as is the case in practice, $V_1 = 0$,

$$Q = (q_{13})_0 V_3 .$$

If now a quantity of electricity $\triangle Q$ is added to the body (1), and a deflection δx of the spot results,

$$Q + \triangle Q = (q_{11})_{\delta x} \delta V_1 + (q_{13})_{\delta x} V_3 \tag{2}$$

$$= (q_{11})_{\delta x} \delta V_1 + (q_{13})_0 V_3 + \left(\frac{\partial q_{13}}{\partial x} \right)_0 V_3 \delta x , \tag{3}$$

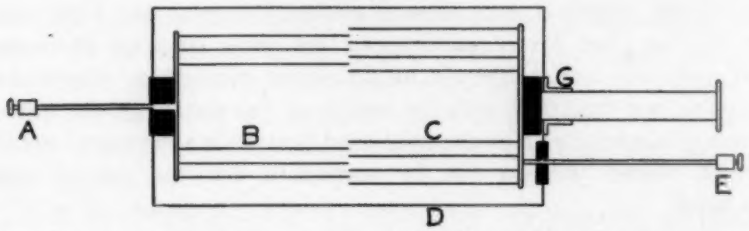
where $(q_{11})_{\delta x}$ is the value of q_{11} for the deflected position δx of the needle. Hence, from (3) and (1) with V_1 zero,[3]

$$Q = q_{11}\delta V_1 + \frac{\partial q_{13}}{\partial x} V_3 \delta x, \qquad (4)$$

where we have omitted the subscript δx for q_{11} and the subscript zero for $\partial q_{13}/\partial x$, since it is sufficiently accurate to regard each of these as corresponding to $\delta x = 0$.

For the measurement of q_{11} and $\partial q_{13}/\partial x$, use was made of a Harms variable condenser, diagrammatically represented in Fig. 6,

FIG. 6.



in which the black parts represent amber. The part C slides into B and, when E is connected to the arbitrary zero of potential, the combined capacity of A, B and the connections may be varied by amounts which are obtainable from a scale reading at G. The outer casing D is connected to the arbitrary zero of potential. In the use of the instrument, A is connected to the insulated quadrant of the electrometer and so to the insulated system of the main apparatus. The quadrant is first connected to the case, and E is raised to a potential U by means of a potentiometer system. This induces upon B a charge $\Delta Q = q_{CB}U$, where q_{CB} is the coefficient of induction between B and C. On releasing B and its connections from the case and connecting E thereto, we obtain

[3] If we write s for $\delta x/\delta V_1$, the potential sensitivity of the electrometer,

$$\Delta Q = \left(q_{11} + \frac{\partial q_{13}}{\partial x} V_3 s\right) \delta V_1 . \qquad (5)$$

If x is in terms of millimetres deflection on the scale, and V_3 is in volts, $\partial q_{13}/\delta x$ is of the order 3×10^{-4} for an electrometer of the small quadrant type, *i.e.*, with a needle about a centimetre in radius. For $V_3 = 50$ volts and $s = 2000$ divisions per volt, the term $\frac{\partial q_{13}}{\partial x} V_3 s$ amounts to 10. It would thus form 25 per cent. of q_{11}. If the electrometer were working near its limit of stability as is frequently the case, a very small increase in V_3 might well double the term, not in virtue of the explicit appearance of V_3 in (5), but by the increase in s resulting from the change of V_3.

an alternation of potential δV and a deflection δx of the electrometer given by

$$\Delta Q = q_{CB}U = (q_{11} + q_{00})\,\delta V + \frac{\partial q_{13}}{\partial x} V_3 \delta x\,, \qquad (6)$$

where q_{00} is the capacity of the condenser BC for the setting used. The quantities q_{CB} and q_{00} are known from the setting of the condenser. By repeating the measurements with different values of U or V_3, or both, we may obtain another equation, and from the two equations q_{11} and $\partial q_{13}/\partial x$ may be calculated, at any rate in theory. In practice, while this method is satisfactory for obtaining q_{11}, it is not satisfactory for obtaining $\partial q_{13}/\partial x$ on account of the small value of this quantity. A better procedure for obtaining the latter quantity is to use, in place of the Harms condenser, the compensating condenser associated with the electrometer, and, moreover, to disconnect the main part of the apparatus from the electrometer. By screwing the portion J (Fig. 5) up to a known distance of about 2 mm. from the plate H, it was possible to obtain a coefficient of induction q_{JH} between J and H which was readily calculable with the accuracy necessary for this small quantity by treating H and the end of J as two parallel plates. By repeating the experiment described in connection with equation (6), with two values of V_3, but with the compensating condenser in place of the Harms condenser, it was possible to eliminate the capacity of the electrometer including the disc H connected to it, and so obtain $\partial q_{13}/\partial x$ in terms of q_{JH}. One could then use this value in connection with the measurements with the Harms condenser to calculate q_{11}. The mean of three determinations of q_{11} agreeing to better than 1 per cent. was $q_{11} = 228$ cm., and the value of $\partial q_{13}/\partial x$ was 24.1×10^{-4} with the understanding that V_3 is measured in electrostatic units, and δx is measured in millimetres deflection on the scale at one metre distance.

RESULTS.

Procedure in Taking Data.—The general procedure in making the measurements was as follows: Starting with the heavy cylinder in the left can, the compensation against battery fluctuations and ionization currents was effected in the manner already described. With the quadrant connected to the case, the reading a_1 of the electrometer was taken, and the electrometer key was raised. One minute later, a second reading a_2 was taken, and five

minutes after this another reading a_3 was obtained. The key was then lowered and another minute later the reading a_4 was obtained. The readings a_2 and a_3 served to determine the apparent charge gain in five minutes, and the readings a_1 and a_4, which determine the zero shift in seven minutes, served to correct for the zero shift during the five minutes corresponding to the deflection from a_2 to a_3. Thus, using the principle and notation involved in equation (4), the apparent quantity of electricity ΔQ collected by the solid cylinder was

$$\Delta Q = \frac{q_{11}}{300}\left\{\delta v_3 - \delta v_2 - \frac{5}{7}(\delta v_4 - \delta v_1)\right\} + \frac{\partial q_{12}}{\partial x} V_3 \left\{a_3 - a_2 - \frac{5}{7}(a_4 - a_1)\right\},$$

where $\delta v_3 - \delta v_2$ and $\delta v_4 - \delta v_1$ are the potential changes in volts corresponding to the scale reading changes $a_2 - a_3$ and $a_4 - a_1$, respectively.

A set of readings (usually five in number) of the above kind was taken, and the electrometer was then calibrated for the range of readings used. The solid cylinder was next removed from the left can, leaving both cans empty, the compensation was readjusted if necessary by use of the small compensating condenser on the electrometer, and the whole procedure, including the recalibration of the electrometer, was repeated. The solid cylinder was then placed in the right can, and the complete set of observations was repeated again. The object of performing the experiment with the cans empty was to allow for any lack of compensation between the two parts of the apparatus, or for any spurious currents; and, apart from mere mechanical action of the weight in producing a gradual distortion of the equilibrium, the difference between the readings with both cans empty and with one can filled with the solid cylinder, should give the rate of acquirement of charge as a result of absorption of radiation by the solid cylinder.

Such a group of three sets of observations (solid cylinder in left can, both cans empty, solid cylinder in right can) was carried out, with insignificant variations in procedure[4] six times at Trondhjem, and once at Kew.

The data are summarized in Table I. The symbols R and L serve to designate whether the solid cylinder was in the right or left can, and E refers to the cases where both cans were empty. The column headed dQ/dt gives the apparent rate of acquirement

[4] At Kew, and in the first group taken at Trondhjem, four sets were taken, as indicated in Table I.

of charge of the system for each individual set of observations a_1, a_2, a_3, a_4, referred to above, and the column headed dQ/dt (mean) gives the corresponding algebraic mean. The column headed "zero move" gives the estimated movements of the electrometer zero during the five-minute interval over which the apparent gains in charge were measured. These movements are expressed as a percentage of the total movements during the five-minute interval. They are recorded merely to enable the reader to gauge the definiteness in meaning of the results obtained. One must not be surprised at the occasional large values of these percentages. For it is obvious that if, in any given set of readings, the charging effect were sensibly zero, an insignificant change in zero might show itself as the equivalent of a very large percentage of the net charging effect during the five-minute interval. Again, in considering the variations in zero and the variations in different values of $\frac{dQ}{dt}$ corresponding to the same disposition of the apparatus, one must observe that our intention is not to attach any fundamental significance to the values of dQ/dt as the measure of a real cosmic effect, but rather to interpret the experiments as emphasizing the smallness of the true charging effect. Working under home conditions it would have been possible to eliminate minor spurious electrometer movements to a much higher degree than is represented by the observations; but, working under conditions remote from one's natural base, with only a limited time for the experiments, with a sensitivity of 5000 millimetres per volt on the electrometer, and a condition of pouring rain for most of the time, zero movements of the order of from one to two millimetres on the scale during a period of seven minutes were regarded as not unsatisfactory.

Table II gives for each group, as designated in Table I, the mean apparent values of dQ/dt corresponding to the condition E, and the excess of the values of mean apparent dQ/dt for the cases R and L over those for the case E. The excesses in question are designated symbolically as $R-E$ and $L-E$, and the algebraic means of $R-E$ and $L-E$ are recorded under the head $(R+L-2E)/2$. The algebraic mean of the values of $(R+L-2E)/2$ for all the groups is given in the last column of Table II and designated as i. They represent the net apparent accumulation of charge per second as a result of the action of the solid cylinder.

TABLE I.

Place.	Date and Time of Starting.	Remarks.	Group.	Disposition of Cylinder.	Zero Change. (Per Cent.)	$\frac{dQ}{dt}$ E. S. U. / Sec. $\times 10^7$.	Mean $\frac{dQ}{dt}$ E. S. U. / Sec. $\times 10^7$.
Kew	July 31, 1925 10:30 A.M.	Rain during part of day.	1	*E*	18	68	
					30	15	24
					35	27	
					0	−12	
	12:00 noon			*R*	4	73	
					19	41	
					16	21	55
					12	111	
					290	29	
	2:00 P.M.			*L*	7	83	
					14	45	54
					0	53	
					10	36	
	4:30 P.M.			*E*	136	5	
					45	38	14
					55	5	
					66	10	
Trondhjem	Aug. 15, 1925 8:55 P.M.	Rained very hard during the morning. On the point of raining during evening when observations were taken. No rain at the conclusion of observations.	2	*E*	42	−14	
					55	−17	
					19	−25	−50
					0	−74	
					10	−97	
					30	−75	
				R	15	−62	
					13	−97	
					154	−24	−70
					11	−104	
					6	−61	

		See this column on preceding page, "Rained very hard", etc.		*E*	5 45 14 6 8	−74 −60 74 −43 −15	−24
	Aug. 16, 1925 12:55 A.M.			*L*	81 95 20 2 42	3 1 68 116 −7	36
Trondhjem	Aug. 16, 1925 11:05 A.M.	Fine but overcast	3	*L*	310 30 13 175 10 5	−47 −19 72 10 100 46	27
	12:45 P.M.	Weak sunshine		*E*	28 18 4 10 278	−79 −89 −94 −86 −10	−72
	2:10 P.M.	Fairly bright		*R*	72 69 17 10 12 12 5	8 −3 24 −119 −72 −87 −70	−46
Trondhjem	5:30 P.M.	Clear, sunny and warm	4	*R*	12 3 4 28 13	−46 −120 −88 19 −28	−53

TABLE I (*Cont'd*).

Place.	Date and Time of Starting	Remarks.	Group.	Disposition of Cylinder.	Zero Change. (Per Cent.)	$\frac{dQ}{dt}$ E. S. U. / Sec. $\times 10^7$.	Mean $\frac{dQ}{dt}$ E. S. U. / Sec. $\times 10^7$.
	6:45 P.M.	Clear, sunny and warm		*E*	15	−35	
					5	−142	
					11	−45	−70
					22	11	
					6	−138	
					10	−74	
	8:00 P.M.	Clear, sunny and warm		*L*	0	−81	
					2	−85	
					46	−10	−48
					54	−28	
					5	−36	
Trondhjem	Aug. 17, 1925 11:15 A.M.	Dull day Drizzle	5	*L*	36	14	
					10	−70	
					17	11	
					11	30	25
					4	162	
					20	−49	
					7	52	
					6	47	
	2:00 P.M.	Raining hard		*E*	5	−73	
					3	−143	−46
					30	19	
					6	−36	
					42	5	
	5:15 P.M.	Raining hard		*R*	12	−74	
					21	−89	
					258	−14	−46
					239	−32	
					28	−21	

Trondhjem	Aug. 17, 1925 8:00 P.M.	Heavy rain	6	*R*	11 4 10 0 10 56 4 5	−103 −126 −90 −145 −45 17 −92 −143	−91
	12:00 midnight	Heavy rain		*E*	12 61 26 2 25	−61 −28 −33 −22 23	−24
	Aug. 18, 1925 1:30 A.M.	Heavy rain		*L*	0 23 18 3 4	−6 −44 34 92 61	27
Trondhjem	Aug. 18, 1925 2:25 A.M.	Raining	7	*L*	22 9 15 53 0	32 66 24 19 −6	27
	10:15 A.M.	Drizzle		*E*	0 5 5 3 7	−24 36 −41 −65 35	−12
	12:00 noon	Drizzle		*R*	22 0 0 66 16	−35 12 9 −4 −22	−8

It is of interest to compare the value of i with the atmospheric electric current I for an area equal to the area of the top of the solid cylinder. The atmospheric electric current density is about -5.5×10^{-7} e.s.u. per sq. cm. per second, and the area of the top of the cylinder is 333 sq. cm. Hence

$$\frac{i}{I} = -\frac{27 \times 10^{-7}}{333 \times 5.5 \times 10^{-7}}$$

$$\frac{i}{I} = -1.5 \times 10^{-2}.$$

In other words, *the experiments indicate that the absolute magnitude of the current absorbed by a solid copper cylinder 20.6 cm. in diameter and 19.3 cm. high is no more than 1.5 per cent. of that which would have been obtained by the complete absorption of a vertical*[5] *corpuscular current of density sufficient to account for*

TABLE II.

E E. S. U./Sec. $\times 10^7$.	$R-E$ E. S. U./Sec. $\times 10^7$.	$L-E$ E. S. U./Sec. $\times 10^7$.	$\frac{R+L-2E}{2}$ E. S. U./Sec. $\times 10^7$.	i E. S. U./Sec. $\times 10^7$.
19	36	35	35	
−37	−33	73	20	
−72	26	99	63	
−70	17	22	20	27
−46	0	71	36	
−24	−67	51	−8	
−12	4	39	22	

the replenishment of the earth's charge. Incidentally, moreover, such small effect as is found in these experiments represents a rate of acquirement of *positive* charge, and is thus of the wrong sign to participate in the replenishment of the earth's charge. Even taking the largest of all the values of $R-E$ and $L-E$, recorded in Table II, viz., the values 99×10^{-7} e.s.u., we find for i a value only 6 per cent. of I. Taking the very largest of all the individual determinations of the apparent dQ/dt, the value 162×10^{-7} for August 17, we obtain a value of i less than 10 per cent. of I, the sign again being contrary to that necessary to contribute to the replenishment of the earth's charge.

The Significance of the Residual Apparent Charging Effect Observed.—It is a significant fact that all of the values of

[5] It is not, of course, implied that the corpuscular current, if it exists, would be entirely vertical. The equivalent vertical current gives a convenient basis for comparison, however.

$(R + L - 2E)/2$ are positive with the exception of one negative value of very small amount. And, only two of the fourteen values for $R - E$ and $L - E$ recorded in Table II are negative. All of the values shown under E in Table II are of the same sign if we omit the value 19. This value was obtained at Kew, all the others being obtained at Trondhjem. It is not unlikely that the disposition of the cans was different in relation to the signs of the potentials applied at Kew from what it was at Trondhjem, as no attempt was made to use the same cans on the left side in the two cases. This would account for the difference in sign between the value of E for Kew and those for Trondhjem. The cans would only compensate as regards ionization current provided that the rate of production of ions in the space between each pair was the same. The volume of the space between one pair of cans was 3300 c.c., so that if we measure a rate of production of ions amounting to 10 ions per c.c. per second, the total ionization current produced by the space between one pair of cans would be 158×10^{-7} e.s.u. The average of the negative values of E in Table II is 44×10^{-7} e.s.u./second, so that these values would be accounted for by a difference in the rates of production of ions on the two sides amounting to 28 per cent. of the total value. Such a difference is not impossible, and in future experiments care would be taken to avoid any such difference by close attention to the exact similarity of the surfaces. The existence of appreciable values of E is of course of no detriment to the experiment so long as the values remain constant for a group of readings.

In the case of the measurements of small currents such as those here concerned, it is a matter of some importance to make certain that the electrometer leakage is not such as materially to affect the rate of deflection of the electrometer. It is easy to develop an expression for the rate of deflection of the electrometer with leakage present in terms of that which would be obtained in the absence of the leakage. This was done, and the leakage was measured under its worst conditions with the resulting conclusion that it had an inappreciable effect upon the measurements.

The difference between the different values of $R - E$ and $L - E$, and between the different values of E is probably to be attributed to accidental causes. While these differences are small in absolute value when taken in the light of the consistency usually obtained in work of this kind, it is felt that they are much larger

than they need be if the experiments were performed under normal laboratory conditions; and the fact that the values of $(R-L+2E)/2$ are preponderantly positive, encourages a repetition of the experiments under the best laboratory conditions to ascertain whether these residual effects represent something of cosmical interest, even though, as the present investigation shows, they are far too small and of the wrong sign to provide for the maintenance of the earth's charge.

The acquirement of positive charge (or rather, loss of negative charge) by the solid copper cylinder does not necessarily present us with an anomalous situation. According to a theory of the origin of the earth's charge propounded by the writer,[6] and independently, later, by von Schweidler,[7] the charging comes about by the ejection of electrons from the air by the penetrating radiation. Those ejected within " striking distance " of the earth enter it, and charge it up until equilibrium is established with the atmospheric-electric conduction current. In the case of a body of thickness great enough to absorb the electrons completely, but not great enough to absorb completely the penetrating radiation, electrons may be shot into the metal over those regions of the surface over which the penetrating radiation enters, and out of the surface over those regions where that radiation emerges. If we should assume that the rate of emission of electrons from an element of volume per unit intensity of the radiation is proportional to the density but otherwise independent of the material, and that, further, the coefficient of absorption of the electrons is proportional to the density and otherwise independent of the material,[8] it would result that, for a case where the penetrating radiation was inappreciably absorbed in the solid cylinder, the number of electrons entering that cylinder would be equal to the number leaving it. Variations in the above assumptions might lead to an excess or deficit of the influx in relation to the efflux of electrons, so that, in some cases, one might realize a charging effect in the positive sense, and in other cases one in the negative sense.

By considering the case where the asymmetry of emission of electrons by the penetrating radiation is not complete, *i.e.*, where

[6] *Phys. Rev.*, 9, pp. 555–557, 1917.

[7] *Akad. Wiss. Wien.*, Berlin, 127, pp. 515–533, 1918.

[8] As a matter of fact, it is only necessary to assume that the *ratio* of the electron emission per unit intensity to the coefficient of absorption for the electrons is independent of the material.

all the electrons are not ejected in the direction of the penetrating radiation, and by taking account of the ejection of corpuscles from the earth to the insulated body as well as from the air, H. Benndorf [9] has developed expressions for the rate of charging of an isolated body such as that used in the experiments here described, and has shown that the charging effect may be positive or negative, depending upon the constants associated with the materials concerned.

A conceivable cause responsible for an insulated body acquiring a charge is of course to be found in a possible spontaneous radio-activity, by which electrons of long range are ejected from the body. Currents of the order of those recorded in Table II would require the ejection of about six electrons per sq. cm. per second. If the electrons were of a range comparable with the dimensions of the cylinder, *i.e.*, of range enormously long compared with the range of ordinary β-rays, we would require the ejection of less than one electron per c.c. per second to account for the currents observed. However, considerations based upon such a possibility are highly speculative.

In conclusion, the writer wishes to acknowledge the assistance rendered by Mr. Oscar Steiner, and Mr. A. M. Enersen, who constructed the apparatus, and Mr. C. A. Kotterman, who assisted in various ways in the tests.

Modern Railroad Signals and Train Control.—Extension of automatic signals and train control devices now being made on the Pennsylvania Railroad involve expenditures totalling $8,000,000. The expenditures being made at this time represent the greatest investment and most extensive installation in signal protection ever undertaken on the Pennsylvania Railroad or any other railroad at one time.

Important new principles to guard against failures in the observance of signals are being worked out by the company in connection with this program. A new device has been designed which consists of electrically operated mechanism by which the indication given by the "wayside" signals—that is, the signals displayed on masts at the side of the track or on overhead signal bridges—is duplicated in miniature within the engine cab, keeping the indications continuously before the engineman and fireman. In addition to the engineman and fireman having continuously before them signals in the cab, each "less favorable" indication by these signals is immediately called

[9] H. BENNDORF, "Über das Grundproblem luftelektrischer Forschung," *Phys. Zeit.*, **26**, pp. 81–87, 1925.

to their attention by an audible warning—a whistle which is connected with the signals. There are two sets of signals in the cabs, one on the engineer's side and the other on the fireman's side. Thus, each will receive an identical separate warning. A train control system, involving these cab signals and what is known as a "stop and forestaller" device, has just been completed on the Pennsylvania Railroad's main line tracks between Harrisburg and Baltimore. This involved the equipping of approximately 150 locomotives with the necessary electrical and mechanical apparatus for the operation of this system, which is actuated by electrical circuits in the track itself.

The present program of the Pennsylvania Railroad for the extension of cab signals and train control covers the main line from Harrisburg to Altoona, the main line from Camden to Atlantic City, the main line of the Panhandle Division from Pittsburgh to Columbus, Ohio, and the main line of the Columbus Division from Columbus to Indianapolis. When the current program is completed approximately 1150 engines will be equipped with the cab signals and other control devices, while the necessary electrical apparatus will be applied to 1530 miles of track.

The "stop and forestaller" is a device so arranged that as a train passes a signal showing any indication except "clear," the air-brakes will be automatically applied unless the engineer "acknowledges" the signal as repeated in the cab by working the "forestaller" controlled by a small lever in the cab. This action "forestalls" the automatic operation of the air-brakes, but of course will not be taken unless the engineer has observed the signal and is therefore informed as to the track conditions ahead, permitting him to bring his train under control.

The electrical system adopted in the installation between Harrisburg and Baltimore is limited to three signal indications in the cab. These are "clear," "approach" and "slow." The new plan which has been worked out, and is known technically as the "coder system," permits four indications, namely, "clear," "approach," "approach-restricting" and "stop." The "approach-restricting" signal is used to show conditions three blocks ahead, for which wayside signals are provided on various portions of the railroad. Four signals permit a more complete and satisfactory repetition in the cab of the "wayside" signals than is possible with only three indications.

Use of Acetylene in Anæsthesia.—The chief uses of acetylene have been as an illuminant and in the synthesis of organic compounds. JULIUS D. GOLDMAN (*Dental Cosmos*, 1926, **68**, 1055–1057) describes the use of acetylene in gas anæsthesia. When anæsthesia is produced by a mixture of nitrous oxide and oxygen, anoxemia or a deficiency in the oxygen content of the blood occurs. When a mixture of nitrous oxide, oxygen, and acetylene is used as the anæsthetic, the oxygen content of the arterial blood is always kept within normal limits. J. S. H.

SUBMARINE INSULATION WITH SPECIAL REFERENCE TO THE USE OF RUBBER.* [1]

BY

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REQUIREMENTS FOR SUBMARINE INSULATION.

THE preëminent requirements for submarine insulation may be grouped under three heads: (1) Mechanical adaptability, (2) suitable specific electrical constants, and (3) permanence, both of mechanical and electrical characteristics. Gutta percha (with or without admixture of balata), which meets these requirements excellently, is regarded as the only reliable material for this purpose. The limited supply and high cost of gutta percha and balata constitute a great handicap on the submarine cable industry. Great caution is necessary in selecting a substitute on account of the large investments involved in a submarine cable and the very high cost of making repairs after the cable is laid. An unusually high degree of security against defects, immediate or future, is therefore imperative. This fact greatly curtails a choice of substitutes. For example, the application of some insulating materials to a wire or cable may often be conveniently made in the form of tapes, wrapped on spirally or laid on the wire longitudinally. Such procedures, however, involve an inherent liability to defect on account of the seam which extends the full length of the wire. Since a seam cannot be relied upon to be perfect for mile after mile, such a method is not commercially wholly desirable. On this account the alternative commercial method is preferred and a continuous sheath of insulation is usually extruded about the conductor. Adaptability to extrusion is therefore an important requirement in a substitute.

Many materials, such as oil gels, factices, mineral rubber substitutes, etc., which suggest themselves as possibilities, are subject to the objection, among others, that the mechanical

* Communicated by Mr. A. R. Kemp.

[1] The literature on submarine insulation lacks, to an extraordinary degree, experimental evidence in sufficient detail to permit intelligent criticism. On this account, no general review of the literature is included in this discussion.

strength is low, or that little is known of their tendency to chemical decay, especially under the conditions existing at sea-bottom. Other equally valid objections exist to most thermoplastics, many of which have been proposed by numerous inventors. A very common difficulty is excessive brittleness or lack of flexibility especially at sea-bottom temperatures. The only reasonable prospect among existing commercial materials as a substitute for gutta percha or balata appears to be some form of rubber. This material can be made to possess an adequate mechanical strength, and in addition it has a long industrial history, during the course of which a vast amount of information has been collected and recorded regarding its durability. Rubber has been used as submarine insulation to a considerable extent for years, especially for short cables or where requirements are less stringent. While little has been made public regarding the electrical deportment of rubber under these conditions, the fact that such cables continue to be manufactured is sufficient evidence that they are not subject to chemical disintegration to a prohibitive degree. Examination of samples of the Seattle-Sitka cable, which had lain sixteen years or more at sea-bottom, has shown that the rubber insulation is still in an excellent state of preservation in respect to its mechanical characteristics. This is not surprising, as the darkness, low temperature and reduced oxygen supply at sea-bottom are conditions favorable to the preservation of rubber.

Fortunately, due to the development of the plantation industry, rubber is available in large quantity and in standardized qualities. In spite of recent contrary tendencies of the market, it can be produced at low cost. Rubber, therefore, meets the only serious objection which exists to the continued use of gutta percha. However, rubber has some distinct technical limitations which will be discussed later.

Considered from a mechanical standpoint alone, gutta percha is a very advantageous material for the manufacture of cable core. It becomes highly plastic at moderately elevated temperatures, so that it may be extruded in a smooth continuous sheath. By the simple process of cooling this sheath sets promptly to a firm, tough and flexible covering which retains these properties at the low temperature of sea-bottom. In addition, whenever a joint or a repair has to be made, the adjacent gutta percha can be softened readily by warming, and fresh material can be molded

into place by the simplest means. Unfortunately, rubber in a pure condition does not have these characteristics. It requires much mechanical mastication, as well as heat, to render it plastic. Even so, in a pure state it still retains too much resilience for smooth working, either by extrusion or calendering. Still more serious is the property of plasticized rubber that it hardens again very slowly and never fully, or sufficiently for our purpose. As is well known, rubber may be more fully plasticized by mixing with it a solid material in a finely divided condition. By this means, satisfactory working qualities can be attained. In order, however, to attain the maximum toughness it is necessary to incorporate sulphur in the mixture, and after forming the insulation in place, to vulcanize it by heating usually at 120° C. to 150° C. The heating must be carefully regulated and uniform throughout the large mass involved in a long length of cable core. This is a difficult matter to accomplish. In the early stages of the heating, the rubber compound softens a great deal, and deformation of the sheath is very liable to occur.

The sulphur in vulcanized rubber is also objectionable, as it promotes corrosion of the conductor, especially during vulcanization. For this reason, a tinned copper conductor is considered necessary, and an additional protection in the form of a sulphur-free layer of insulating material next to the conductor is desirable. Even so, a bimetallic conductor, such as tinned copper, is probably not as safe as a monometallic one, as there is some danger of electrolysis between the two metals, leading to corrosion. This is especially true in shallow water where the oxygen content of the water is relatively high. However, this danger is lessened by the outer coverings of a cable which probably greatly reduce the diffusion of oxygen into the core. For the greater length of ocean cables in water averaging a mile or more in depth, the diminished supply of oxygen makes this point relatively unimportant.

In spite of the above-mentioned handicaps, a mechanically satisfactory core can be made of vulcanized rubber. The quantity of product which has to be discarded on account of defects is likely to be larger and the labor and plant charges are likely to be heavier than in the manufacture of gutta percha core. These increases in cost fortunately may be more than offset by the lower cost of the raw material for rubber cores.

Vulcanized rubber is superior to gutta percha in certain minor respects, such as resistance to oxidation and to local heating which may occur during manufacture of the cable. These factors are of some importance in connection with the application of lead sheath to cable core when desired, and also in the cost of transportation and storage of cable.

In either a telephone or a telegraph cable, a low capacity is the electrical characteristic of first importance. In cable practice heavy walled insulations are commonly used, in order to lower the electrostatic capacity of the cable and to permit insurance against local faults by application of the insulation in multiple layers. Consequently the specific insulation resistance of the material need not be held to a high standard in order to meet all requirements. The breakdown strength of the insulation, if it is otherwise satisfactory, is also always in excess of the requirements. The leakance or conductance [2] becomes a factor of considerable importance in telephone cables, and especially so with high-frequency alternating currents, notably the so-called "carrier" currents which are coming into greater use constantly.

Aside from mechanical adaptability to manufacture and the absolute values of dielectric constant and phase difference angle,[2] the most important considerations are stability and uniformity of properties. From the standpoint of permanence of mechanical and electrical characteristics gutta percha has proven to be nearly ideal, there being practically no change over a period of fifty years [3] at sea-bottom. All evidence which is available indicates

[2] In cable engineering practice the term "insulation resistance" refers to the resistance offered by the material to the flow of a direct current. The term "conductance" or "leakance" means the reciprocal of the resistance of the material to an alternating current. To be significant the frequency of alternation must be stated since the value of conductance is for most materials approximately proportional to the frequency. We have chosen to use phase difference angle as an expression of this property since this term is probably somewhat more familiar to the general reader. The phase difference angle in degrees $= \frac{57.3G}{2\pi FC}$ in which 57.3 is the conversion factor for radians to degrees, G the reciprocal of the measured resistance in micro-ohms, F the frequency in cycles per second and C the capacity in microfarads. The power factor as used by cable engineers is, in the same terms, $\frac{G}{2\pi FC}$.

[3] Actually it is not practicable, except in short cables, to measure electrical characteristics with great exactitude. Judgments such as this are therefore based largely upon operational functioning.

that vulcanized rubber as heretofore manufactured is equally permanent in its mechanical properties under the same conditions, but is lacking in stability of its electrical properties. This instability in electrical properties is manifested in rubber insulated cables principally by slow increase in capacity over periods of years. In some cases the dielectric constant of the insulation has become double what it was at the time of manufacture. The other electrical properties are also adversely affected in some cases. These changes are largely due to penetration of water.

MOISTURE RESISTANCE OF ORGANIC SUBSTANCES AS AFFECTING ELECTRICAL PROPERTIES.

In discussing insulating materials, it is necessary to distinguish sharply between those properties which are inherent in the material and those which are dependent upon the mechanical or physical condition of the material or the presence of impurities, notably moisture. As is well-known, paper or textiles when dry are good insulators, but very poor ones when wet. To a much lesser degree this is also true of gutta percha, rubber and other plastics. The insulation engineer must not count on anything being " water-proof " in the sense of being unaffected electrically by moisture. Certainly moisture penetration through such dense materials as metals is very slow indeed; but from experience with a large number of substances it is very probable that all organic materials are measurably affected electrically throughout their mass by moisture penetration. This phenomenon cannot be too strongly emphasized.

Gutta percha, which must be taken as the criterion of suitability for submarine insulation, is also subject to the effects of moisture. It is ordinarily prepared for application to a conductor by a process of washing to remove mechanical impurities, and a subsequent kneading operation during which the material is partially dried. When applied to the conductor, the mixture has somewhat the following composition:

	Per Cent.
Gutta hydrocarbon	50–60
Gutta resins	40–50
Fine dirt, bark, etc.	1–3
Water	1–3

In good cable practice the water content is held fairly closely to the limits 1.5 to 2.0 per cent. There are two good reasons for

this: One is that the working incident to further removal of water promotes chemical deterioration of the gutta percha; the other is that by experience gutta percha experts have found that the cable cores change very little in electrical characteristics during subsequent immersion in water if they are made with an initial water content not far from that which they will ultimately attain after years in the sea.

The electrical characteristics of such cable mixtures can be varied considerably by proper choice of the different varieties of gutta percha and by suitable proportioning as well as by governing the water content. In a very general way the cheaper, more highly resinous materials commonly possess high insulation resistance and rather high dielectric constants. Conversely, the tougher, less resinous gutta perchas have lower electrostatic capacities and insulation resistances. However, numerous exceptions to these statements can be found. A small percentage of rubber or bitumen is sometimes added to gutta percha mixtures to lower the capacity. Additions of balata are commonly resorted to when a small phase angle is especially desirable. The electrical characteristics are also sometimes brought within specification limits by a greater or less degree of mastication which affects the immediate electrical characteristics chiefly by controlling the moisture content. To a great extent this practice is therefore a form of juggling that influences the characteristics of the freshly made core far more than those which will be attained after the cable has lain some years at sea-bottom.[3] The electrical characteristics at 25° C. of gutta percha insulation so prepared are usually about as follows:

Dielectric constant, 1000 C.P.S.—3.1–3.5.
Phase difference angle, 1000 C.P.S.—10 to 50 minutes.
Insulation resistance, 5×10^3 to 10^9 megohms per cm.^{3}

Such values refer to commercial cable mixtures when applied to the conductor and are naturally affected by the frequency, temperature and pressure at which the measurements are made. Each of these factors is of sufficient consequence to require consideration in the design of a cable for a given operating frequency, depth and temperature of water. In general the capacity is slightly decreased while the phase difference angle is largely increased by increase of frequency. Lowering of temperature slightly increases capacity. In the case of gutta percha the phase

difference angle rises sharply as the temperature falls from 25° to 0° C. Pressures up to 10,000 pounds per square inch slightly raise capacity and may as much as double the phase difference angle. Both temperature and pressure effects on capacity correspond quite closely to their effects on the density of the material. Practically the effect of these factors on insulation resistance does not need to be considered as the insulation, if intact, always exceeds requirements in this respect. However, the resistances are much higher at sea-bottom than the values above cited. The values of frequency, temperature and pressure at which the electrical measurements reported in this paper were made are arbitrarily chosen in harmony with general cable works practice.

In addition to being dependent on the factors mentioned above, the electrical characteristics of gutta percha change with

TABLE I.

Electrical Characteristics of Wet and Dry Gutta Percha.

Condition of Tests.	Water Content, per Cent.	Dielectric Constant.	Phase Difference Angle Minutes.
		1000 C.P.C. at 75° F.	
Freshly made	2.32	3.14	14
1 month in sea water	1.81	3.08	16
33 months in sea water	1.35	3.06	16
6 months in sea water	1.06	3.03	14
Dried in vacuum over calcium oxide	.00	2.85	15
Reimmersed for 1 month in sea water	0.85	3.06	16
Reimmersed for 3 months in sea water	0.88	3.07	16

subsequent drying and wetting. Such variations are avoided by the manufacturer as far as possible. As an example of the effect of water content on the electrical properties of gutta percha, the data in Table I are given. The values were obtained on a sample of commercial gutta percha in the form of a sheet approximately .050 inch thick. Similar changes have been observed in gutta percha cable core produced by well-recognized manufacturers.

Rubber insulation shows the same sort of changes in water content and in electrical values with wetting and drying. However, the magnitude of the changes in most rubber insulation is very much larger than in well-prepared gutta percha. Rubber compounds as prepared for vulcanizing are necessarily quite dry. After vulcanizing under steam pressure the water content usually is .2 to .5 per cent.; vulcanization under water produces a still wetter rubber, or vulcanization in a dry atmosphere, an almost

perfectly dry rubber. Since electrical properties depend in part on water content, it is necessary to compare the specific properties of insulating materials on some common bases, preferably in a dry state as well as under conditions of use, in order to gain an understanding of the controlling factors. On a dry basis unfilled vulcanized rubber is comparable with gutta percha in dielectric constant and phase difference angle. On the other hand, after prolonged immersion in water, rubber commonly compares quite unfavorably with gutta percha. The effects of frequency, pressure and temperature are similar for rubber and gutta percha except that pressure, and particularly temperature, affect the phase difference angle of rubber much less.

EXPERIMENTAL PROCEDURE IN THE STUDY.

In order to evaluate materials for submarine insulation, a method was devised to give a reasonably rapid approximation of the effects of sea-bottom conditions on such insulations. Thin sheets of the various materials were prepared and their water content and electrical characteristics measured both dry and after various periods of immersion in aqueous media. In order to permit analysis of the various factors involved, it seemed desirable to study the materials first in simple form, rather than complex mixtures such as are often used for rubber insulation. The results obtained with sheet samples have in the main been verified by preparation and testing of cable core samples. While room temperature and atmospheric pressure were used in most observations, variations of these factors were also studied separately.

The wisdom of the choice of thin sheets was amply justified by the event which demonstrated that an approximate result could be attained in this way in a few weeks which would require months or years in full-size cable core samples, which usually have a conductor diameter of .1 to .2 inch and an outside diameter of .3 to .5 inch. Andrews and Johnston[4] have mathematically analyzed the effect of dimensions on the rapidity of saturation of rubber masses by water for the purpose of predicting when substantial saturation will occur in masses so thick that direct experimental determination is impractical. Our results are in conformity with their conclusions. However, in general, it is an adequate approximation to say that the time required for a

[4] *Jour. Am. Chem. Soc.*, 46, 640 (1924).

definite rather high percentage of saturation of sheets of a given compound is roughly proportional to the square of the thickness. For small degrees of saturation neither Andrews and Johnston's formula nor the above approximation agrees well with the facts. Clearly the amount of water absorbed during the first moment of exposure to water is independent of the thickness and depends only on the surface exposed. This fact is not taken into account in the development of these formulæ. Similar principles, of course, hold for other shapes, such as cores.

Also, a method of electrically testing sheets was developed and applied quite generally, as some correlation of water content and electrical characteristics was to be anticipated. Essentially the method consists of placing a sheet of rubber of known thickness between two accurately plane-surfaced steel electrodes, and measuring the capacity and conductance at 1000 c.p.s. with a suitable bridge. Direct current insulation resistance is also measured with a sensitive galvanometer. The changes in weight of the rubber sheets are noted as an index of the water content from time to time. A system carefully standardized so as to be uniform is necessary for surface cleaning and drying to avoid anomalies due to surface contact between electrode and insulating material. The sheets are rinsed with distilled water, blotted with filter paper, exposed to a current of dry air for one minute, and immediately placed between the electrodes, readings being taken after one minute.

MECHANISM OF WATER ABSORPTION.

Postponing consideration of the electrical results for the moment, we will describe some results of study of the water absorption of gutta percha and some simple forms of rubber sheets. The fact worthy of first mention is that the amount of water absorbed by sheets of uniform thickness of a given material depends on the salt content of the water in which the sample is immersed. Table II gives a few selected but typical observations. Indications of this had already been noted by Obach [5] in the case of gutta percha, although he offered no explanation of the "curious results." The facts suggested that water absorption is dependent on the osmotic pressure of the external solution, and

[5] "Cantor Lectures on Gutta Percha," Appendix VIII, William Trounce, London (1898).

led to the corollary proposition that the amount of water absorbed is also controlled in part by the molecular concentration of water-soluble substances contained within the mass, *i.e.*, the osmotic pressure of the internal solution. This viewpoint can be readily illustrated by milling into a rubber mass a small amount of salt and immersing the sheet in fresh water. In a few weeks the position of the salt particles is marked by water-filled blisters which protrude from the surfaces of the rubber and often burst from the internal osmotic pressure which is developed. The effect

TABLE II.

*Relation between Water Absorbed and Salt Content of Water in Which Sheets of Gutta Percha and Rubber Are Immersed.**

Sample.	Per Cent. Water Absorbed at Saturation at 25° C. Distilled Water.	3.5 per Cent. NaCl Solution.	10 per Cent. NaCl Solution.	Saturated NaCl Solution.
Vapor pressure of solution mm. Hg	23.6	23.1	21.9	17.9
Gutta percha (Pahang)	> 4.0	1.5	1.2	.9
Soft vulcanized washed smoked sheet	> 14	1.0	0.6	.3
Soft vulcanized unwashed smoked sheet	> 20	3.2	1.0	.5
Raw rubber washed smoked sheet	> 14	1.1	.7	.4
Raw rubber S.S. unwashed	> 117	4.4	1.6	.6
Rubber hydrocarbon†		.5	.4	.3
Vulcanized compound washed rubber and silica	> 1.4	.4	.2	.1

*In the case of the rubber sheets it was necessary to prevent access of air to the solutions for exact results.

†This rubber was purified by dissolving acetone extracted first latex crepe rubber at room temperature in petroleum ether B. pt. 40°–60° C., separating the insoluble matter by decantation and precipitating the hydrocarbon from solution with alcohol. The hydrocarbon was then freed from alcohol and solvent by heating at 50° C. under vacuum. The product was clear and practically colorless. It was free from ash and contained .003 per cent. nitrogen. The original acetone extracted pale crepe contained .42 per cent. nitrogen.

of the water-soluble constituents normally present in rubber is also to cause increased water absorption, but in this case on account of the higher degree of dispersion no visible blisters appear. Removal of water-soluble substances, especially those of crystalloidal character, by thorough washing produces substantial reduction in the water absorption, as is also shown in Table II. Further information on this point is found in Tables III and IV. The hygroscopic character of the water-soluble substances in rubber has been demonstrated by Whitby.[6]

It will be noted in Table II that soft vulcanized rubber shows this osmotic effect as markedly as raw rubber. We have reason to believe that the phenomenon of osmosis plays a rôle of con-

[6] *J.S.C.I.*, 37, 278T (1918).

siderable importance in determining the extent of water absorption by organic plastics generally. That water absorption invariably decreases with increase of salt (sodium chloride) concentration in the external solution has been noted in about 1200 rubber samples, both raw and vulcanized, filled and unfilled, hard and soft.

There is indeed no strict proportionality between osmotic pressure of the external solution and the extent of water absorption by rubber. None is to be expected in view of the fact that

TABLE III.

Effect of Washing on Absorption of Water by 50 Mil Sheets of Raw Rubber Immersed in 3.5 per Cent. Salt Solution.

Description.	Time of Soaking.*	Time of Washing.*	Water Absorbed (Per Cent.).	Time to Reach Approx. Saturation (Days).	Remarks.
Sprayed rubber.....	None	None	> 28.0	> 58	900 parts per million total solids in wash water for all except last sample.
Smoked sheet.....	None	None	6.8	180	
Smoked sheet, Lot 1.	None	5	1.3	15	
Smoked sheet, Lot 2.	2 days	3	1.2	14	
Smoked sheet, Lot 3.	2 weeks	1	2.9	28	Taken from a uniform lot of rubber.
Smoked sheet, Lot 3.	None	None	6.3	..	
Smoked sheet, Lot 4.	1 week	2	2.9	30	
Pale crepe, Lot 5....	2 weeks	5	1.2	12	Taken from a uniform lot of rubber.
Pale crepe, Lot 6....	2 weeks	24	1.1	10	
Pale crepe.....	None	None	> 7.2	> 90	
Smoked sheet, Lot 7.	None	5	.7	10	150 parts per million total solids in wash water.

*The sheets of smoked rubber were individually passed through corrugated rolls and laid loosely in a tank of cold fresh water for the periods indicated. The crepe was also similarly soaked without previous treatment. Washing in all cases was carried out in a continuous type of washer with corrugated rolls under a stream of water at about 65° C.

within the rubber are present a variety of substances of inaccurately known molar concentration, of wide range of solubility in water and of temperature coefficients of solubility and of all degrees of diffusibility through rubber. Some of the substances naturally present in rubber are capable of outward diffusion, as shown by the fact that the water content, as determined by drying a piece of plantation rubber which has been soaked for years, is greater than the weight increase the rubber has undergone. Further, the extracted matter can readily be found in the external liquid by analysis. Indeed, it is a common phenomenon that the

weight of a piece of rubber soaked in water for months rises to a maximum and later declines materially as the extractive matter slowly diffuses outward. Only in case the rubber is permeable to water alone and not to any solute present will the balance in osmotic pressures of internal and external fluids be adjusted by interchange of water alone.

However, there are other complications. The purest rubber obtainable absorbs a limited amount of water when immersed in salt solutions. The more nearly pure is the rubber, the more

TABLE IV.

Effect of Washing Raw Rubber on Water Extract and Water Absorption from 3.5 per Cent. NaCl Solution.

Sample.	Time of Washing Hours.	Water Extract, per Cent.*	Per Cent. Water Absorbed.	
			14 Days' Immersion.	26 Days' Immersion.
Smoked sheet A.....	0	0.57	5.0	5.6
Smoked sheet B.....	0	0.23	2.9	3.5
Smoked sheet B.....	1	0.11	2.4	2.8
Smoked sheet B.....	2	0.05	1.9	2.1
Smoked sheet B.....	3	0.03	1.2	1.3

*By a modification of method of A. Van Rossem, Comm. Delft II, 406, 1917, C. A. 12, 639, 1918.

nearly exact is the correspondence between the water absorption and the aqueous vapor pressure of the solution in which it is immersed. For salt solutions of higher concentration (*i.e.*, lower vapor pressures), the correspondence is quite close, even with commercial rubbers, but at lower concentrations there is a substantial discrepancy. If the vapor pressures of the solutions are plotted against the water content of the rubber samples at equilibrium, a straight line results over the lower range of vapor pressures, breaking rather sharply at higher vapor pressures. Briefly, at low vapor pressures rubber obeys Henry's law and therefore appears to dissolve some water to form a true solution. That it takes up a further and usually larger quantity by osmosis is indicated by the divergence from Henry's law at higher vapor pressures. The reader is referred to a paper by Lowry and Kohman [7] for a fuller discussion.

Porosity of the rubber may also play a part in water absorption, though this is apparently not the case in well-made rubber sheets containing no considerable amount of filler. If such pores

[7] Shortly to be submitted for publication in the *Jour. Phys. Chem.*

exist and are open to the surfaces, the rubber will absorb salt as well as water, which we do not find to be the case. If the pores exist as closed pockets, selective absorption of water will occur, but the distribution of the water will be likely to be non-uniform and to reflect itself in non-uniformity of electrical characteristics in different portions of the mass, which is also contrary to our general experience. However, such non-uniformity has been detected by electrical measurements in certain sheets which were undoubtedly porous.[8] Likewise, porosity would be likely to be affected by methods of working. We do not find methods of working of serious consequence in affecting water absorption. Other pertinent evidence is offered by Lowry and Kohman.

Lastly, the presence of proteins in rubber has a distinct influence on water absorption though the magnitude of this influence is certainly small as compared with those osmotic effects

[8] Air-bubbles frequently exist in raw stocks which will result in porosity in the cured material unless avoided by precautions in vulcanizing. Porosity as produced by gas expansion is influenced greatly by sudden and large changes of pressure at certain stages of vulcanization. In order to secure a picture of such changes of pressure in pores, the following experiment was performed. A sheet of uncured compound ¼ inch thick was perforated with several circular holes ¾ inch in diameter. The perforated sheet was then laid between two non-perforated sheets of the same rubber stock .050 inch thick and the whole rolled down so that good adhesion of the faces of the sheets was obtained, leaving cylindrical air spaces in the interior about ¼ × ¾ inch diameter. The mass was then carefully bedded in talc and placed in a vulcanizer to which steam was admitted to a pressure of 40 pounds. After one and one-half hours the steam pressure was allowed to decline slowly over the space of one-half hour before the vulcanizer was opened. The air spaces in the interior were found to have collapsed nearly completely. However, by puncturing the wall of a cell and thus admitting air, it was found that the interior surfaces were not adhering. On the contrary, both the upper and lower diaphragms of the cell sprang outward to form a convex, instead of a concave, surface as soon as air was admitted. By putting such an unpunctured cell in a vacuum desiccator and slowly evacuating till the diaphragms sprang outward to a convex surface, the internal gas pressure in these cells was found to be diminished below atmospheric pressure by approximately 25 inches of mercury. The history of the changes in internal pressure was very clearly shown. At first the air in the cells had expanded, swelling out the diaphragms and it was in this position that vulcanization took place, which accounts for the non-adherence of the diaphragms and their permanent set in convex form. While in this position, most of the air in the cells diffused out through the rubber. Later on cooling in the vulcanizer, the diaphragms collapsed, but being vulcanized did not adhere to one another.

which may be produced by crystalloids. It has been experimentally shown that rubber freed from nitrogen as completely as possible absorbs a substantial fraction of the amount of water absorbed by ordinary rubber which has been subjected to a thorough washing. (See "Rubber Hydrocarbon" in Table II.) However, the influence of proteins on water absorption, and particularly upon the electrical stability of rubber in water, requires further study. This work has not progressed far enough to permit a full discussion at this time.

LIMITS OF WATER ABSORPTION.

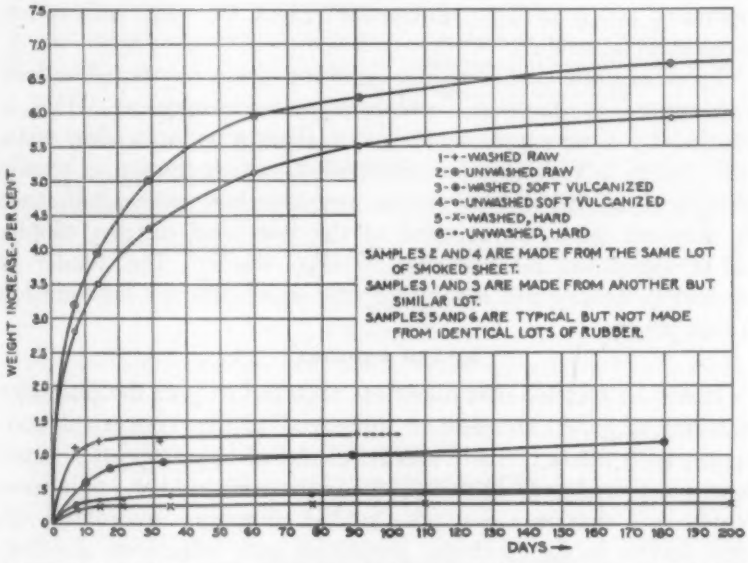
It has been reported [9] that raw rubber will eventually become "practically dispersed" in distilled water at higher temperatures. We have made no strictly comparative observations. At lower temperatures complete dispersion has not been observed. One sample of raw smoked sheet was observed to absorb 70 per cent. of its own weight after thirteen months' immersion in distilled water. A sample of "sprayed rubber" of commercial origin, said to contain all the natural serum constituents, absorbed 348 per cent. of its own weight after a year's immersion in distilled water. In each case absorption of water had not entirely ceased at the end of the periods indicated. Both samples, though soft, sticky and weak, were still coherent sheets. Well-washed raw rubber absorbs much less water during a given period of immersion in distilled water, but the absorption is extended over many months and in our experience reaches no definite limit, though it appears to approach one asymptotically. In oxygen-free salt solutions, both washed and unwashed rubbers reach definite limits of water absorption. After long periods of soaking in either pure water or salt solutions, especially those of moderate concentration, rubber sheets of washed and unwashed raw material present a strong contrast in appearance. The former are little changed in appearance by soaking, remaining translucent and but little altered in consistency or dimensions. The latter acquire a gray or nearly white opaque appearance. Both undergo a change in dimensions corresponding in volume to the volume of water absorbed. Circular sheets of rubber originally 5 inches in diameter have been observed to increase to a diameter somewhat over 7 inches.

When we turn to soft vulcanized rubber, we find the situation

[9] BOGGS and BLAKE, *Ind. and Eng. Chem.*, **18**, p. 224 (1926).

but little changed, although there is a general tendency to approach limits of absorption in distilled water somewhat more promptly and definitely. This is attributable in our view to increased resistance to deformation as a result of vulcanization. This is further borne out by the deportment of hard rubber which offers still greater resistance to deformation by swelling. In this case the water absorption is very little affected by the amount of water-soluble matter present in the original rubber

CHART I.


WEIGHT INCREASE-PER CENT
DAYS
1-+-WASHED RAW
2-●-UNWASHED RAW
3-●-WASHED SOFT VULCANIZED
4-○-UNWASHED SOFT VULCANIZED
5-X-WASHED, HARD
6-••-UNWASHED, HARD
SAMPLES 2 AND 4 ARE MADE FROM THE SAME LOT OF SMOKED SHEET.
SAMPLES 1 AND 3 ARE MADE FROM ANOTHER BUT SIMILAR LOT.
SAMPLES 5 AND 6 ARE TYPICAL BUT NOT MADE FROM IDENTICAL LOTS OF RUBBER.

Progress of water absorption at room temperature of unfilled rubber in 3.5 per cent. NaCl solution as affected by washing and by vulcanizing.

(Chart I). If one imagines a rigid, closed vessel, completely filled with a solution such that the walls of the vessel are impermeable to the solute while permeable to the solvent, it is readily seen that osmotic exchange, which begins on immersion of the vessel in pure solvent, will soon cease on account of the hydrostatic pressure produced in the vessel. An approach to this condition is found in hard rubber. However, it is quite possible that other factors contribute to the low water absorption of hard rubber. Chart I shows the progress of water absorption in raw rubber, both washed and unwashed, and in vulcanized samples,

both soft and hard, prepared from both washed and unwashed rubber. Soft vulcanized sheets after long soaking present similar, though less marked, contrasts in appearance between washed and unwashed material than is the case with raw rubber. Opacity often results in washed vulcanized sheets from blooming but on removal of the film of sulphur so developed the interior is found to maintain its translucency, while the unwashed material develops internal opacity. Such changes in appearance are much less marked in concentrated salt solutions. None of these facts is out of harmony with the view that osmosis is a predominantly important factor in determining the extent of water absorption by commercial rubber.

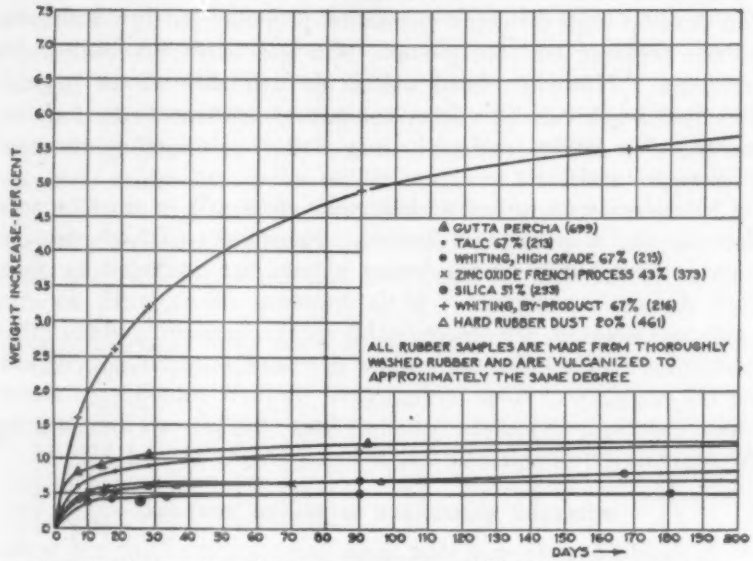
In vulcanized rubber aging, resulting from oxygen dissolved in the water, produces an increase in water absorption. This is reflected by a very prolonged water absorption at a slow rate. This change is very slight in compounds which change in tensile strength slowly in the air and in any case has little significance for deep-sea cable, on account of the restricted oxygen supply, low temperature and darkness in deep water. The reader is referred to Lowry and Kohman [10] for more detailed information on this point.

FILLED RUBBERS.

It will be recalled that fillers are required to give the plasticity necessary to good extrusion of rubber. Our first concern in connection with fillers in their specific electrical characteristics, since these will be reflected in their compounds with rubber. We have already said that in a dry state rubber compares favorably with gutta percha in the electrical properties with which we are concerned. Certain fusible hydrocarbons, waxes, pitches and resins also have similar electrical characteristics. However, such fusible organic materials, though possibly serviceable for admixture in raw rubber compounds, are not generally adapted to admixture in considerable proportions with rubber which is to be vulcanized, as these materials largely defeat the object of the vulcanization, that is, the production of a tough substance. Chemical decomposition also commonly occurs when such mixtures are heated. Of infusible organic materials the choice is very limited. Phenol condensation products, polymerized oil gels and hard rubber may be mentioned. Phenol condensation products on immersion in

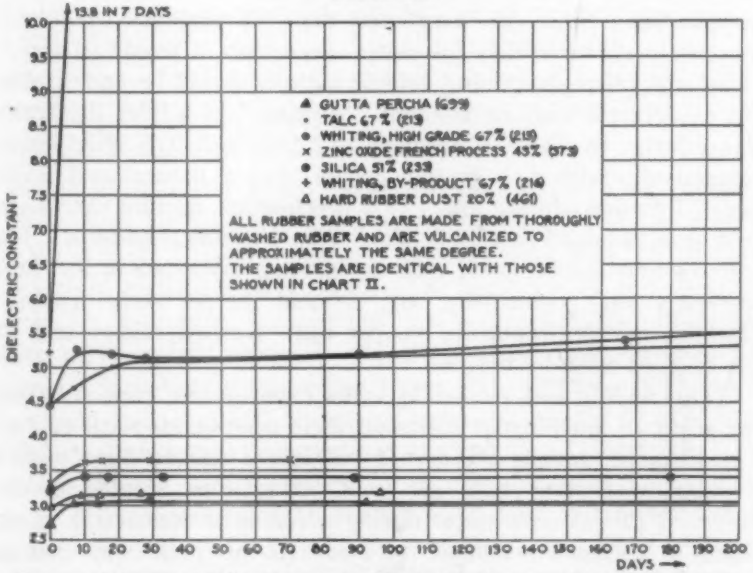
[10] *Loc. cit.*

CHART II.


WEIGHT INCREASE - PER CENT
DAYS
▲ GUTTA PERCHA (699)
• TALC 67% (213)
● WHITING, HIGH GRADE 67% (215)
× ZINC OXIDE FRENCH PROCESS 43% (379)
● SILICA 51% (233)
+ WHITING, BY-PRODUCT 67% (216)
△ HARD RUBBER DUST 20% (461)
ALL RUBBER SAMPLES ARE MADE FROM THOROUGHLY WASHED RUBBER AND ARE VULCANIZED TO APPROXIMATELY THE SAME DEGREE

Progress of water absorption at room temperature of filled rubbers in 3.5 per cent. NaCl solution.

CHART III.


13.8 IN 7 DAYS
DIELECTRIC CONSTANT
DAYS
▲ GUTTA PERCHA (699)
• TALC 67% (213)
● WHITING, HIGH GRADE 67% (215)
× ZINC OXIDE FRENCH PROCESS 43% (379)
● SILICA 51% (233)
+ WHITING, BY-PRODUCT 67% (216)
△ HARD RUBBER DUST 20% (461)
ALL RUBBER SAMPLES ARE MADE FROM THOROUGHLY WASHED RUBBER AND ARE VULCANIZED TO APPROXIMATELY THE SAME DEGREE.
THE SAMPLES ARE IDENTICAL WITH THOSE SHOWN IN CHART II.

Progress of changes of dielectric constant of filled rubber immersed in 3.5 per cent. NaCl solution at room temperature.

water have been observed to absorb several per cent. of water and develop high dielectric constants, probably partly on account of the presence of free phenol. Oil gels have an undesirable resilience. However, hard rubber is available as an organic diluent for vulcanizable rubber compounds and meets the requirements of a filling material in a fairly satisfactory manner. (Charts II and III.)

If attention is turned to inorganic materials, it may be seen that the choice is likewise limited. Substances of high density, such as compounds of the heavy metals, are excluded by their high dielectric constants. Most common fillers, such as zinc oxide and whiting, are objectionable on this account. Many other substances are excluded on account of chemical reactivity (including the tendency to form hydrates) or by their solubility in water. Silica, especially in a non-crystalline state, has proven to be among the least objectionable mineral fillers (Charts II and III).[11]

MOISTURE RESISTANCE OF FILLED RUBBERS.

It should first be said that those factors controlling the water absorption of unfilled rubbers are also operative in the case of filled rubbers. Other new factors are also encountered in filled compounds. Since we have seen that the presence of soluble matters in rubber makes for water absorption, it would be anticipated that fillers containing soluble matters would be undesirable. An experiment with commercial " whiting " as a filler illustrates this effect. In Table V samples 214, 215 and 216 were made, respectively, with (1) ground calcite, (2) a natural and much finer Cliffstone whiting and (3) a precipitated calcium carbonate, said to be obtained as a by-product from a chemical process. The water-soluble matter other than the calcium carbonate in the three materials was, respectively, .07, .07 and .74 per cent. In each case the samples contained 30 per cent. washed rubber and 67 per cent. of filler by weight.

This finding was verified by other experiments, such as noting the effect of washing of fillers on their compounds with rubber. For instance, sample 26, a soft vulcanized rubber filled with a whiting containing 0.40 per cent. extraneous soluble matter absorbed 4.0 per cent. water during ten months' immersion in sea water, a companion sample 33 made in the same way, but in

[11] U. S. Patent No. 1,438,735, Dec. 12, 1922, R. R. Williams.

which the whiting was first washed with distilled water and dried, absorbed only 1.9 per cent. The electrical changes of No. 26 were similar to those of the 216 sample in Table V, while those in No. 33 were similar to those of No. 214.

It is obvious that any chemical reactivity in submarine insulation under conditions of its use is to be avoided. A form of chemical reactivity of the milder sort and therefore more likely to be overlooked is hydration. Talc has the property of hydrating slowly when immersed in water at ordinary temperatures. Two grams of talc which had been dried at 105° C. were covered

TABLE V.

Changes in Whiting Filled Rubber on Immersion in 35 per Cent. Salt Solution. All Electrical Measurements at 75° F.; A.C. Measurements at 1000 C.P.S.

	Sample 214.				Sample 215.				Sample 216.			
Days Immersed.	H_2O Absorbed, %.	K.	Phase Difference Min.	Ins. Res. Meg. per cm.³.	H_2O Absorbed, %.	K.	Phase Difference Min.	Ins. Res. Meg. per cm.³.	H_2O Absorbed, %.	K.	Phase Difference Min.	Ins. Res. Meg. per cm.³.
0	.0	4.4	12	$> 10^9$†	.0	4.5	16	$> 10^9$†	0	5.2	50	10^9
7	.50	4.9	22	6×10^8	.45	5.1	57	3×10^8	1.6	13.8	540	10^4
18	.66	4.9	27	10^9	.52	5.1	59	10^9	2.6	> 15†	−†	<1‡
28	.77	4.9	30	6×10^8	.62	5.1	61	4×10^8	3.2	> 15	−	<1
90	.96	4.9	32	10^9	.71	5.2	71	10^9	4.9	> 15	−	<1
167	1.05	4.9	34	6×10^8	.71	5.4	103	10^9	5.5	> 15	−	<1
761	−*	9.1	630	<1‡	−*	6.2	215	4×10^7	7.0	> 15	−	<1

* Samples lost weight due to blooming.
† Too high to measure accurately with available equipment.
‡ Values <1 megohm were not measured accurately.

with distilled water and allowed to stand at room temperature. From time to time the water was evaporated and the residue dried at 105° C. and weighed. In forty-eight days the increase in weight reached 2.16 per cent. The hydration of talc evidently goes on in rubber compounds, for a gradual and prolonged increase in weight occurs in talc compounds which we have prepared. Sample 213, containing 30 per cent. washed rubber and 67 per cent. talc, was still absorbing water at the end of 167 days, when the percentage of water had reached 1.17 (Chart II). The rubber present would account for an absorption of only .40 per cent. and constant weight should have been attained in about ninety days, as judged from other experiments. The dielectric constant continued to rise with the water content.

Another important feature of filled rubbers is the size of particles of the fillers. It is a well-known fact that fineness of fillers increases the mechanical strength of rubber compounds. Fineness of fillers might be expected to increase water absorption and decrease electrical stability in water because the area of interface between rubber and filler increases with increasing fineness. That this is true of silica is shown by the data in Table VI. In many filled rubbers this interface appears to be the greatest source of electrical weakness. Since few fillers are available in a variety of finenesses and otherwise of uniform quality, we have limited our experiments in this particular to silica.

Schippel [12] noted that when filled rubber is put under strain

TABLE VI.

Properties of 60 per Cent. Silica Rubber Compounds as Affected by Fineness of Filler.

Sample No.	Fineness.	After 80 Days' Immersion in 3.5 per Cent. Salt Solution.	
		Capacity Increase. Per Cent.	Water Absorption. Per Cent.
253D	200 mesh	10	.77
253D$_1$	200 mesh	7	.77
254D	200 mesh	6	.72
254D$_1$	200 mesh	6	.71
255D	One week additional grinding of same filler in ball mill.	26	.92
255D$_1$		13	.93
256D		16	.93
256D$_1$		17	.96

it increases in volume, and advanced the theory that conical vacua are created about the filler particles with their apices in the direction of strain. Green [13] later was able to demonstrate the existence of such vacua by microscopic examination. Certainly the adhesion of rubber to mineral particles is commonly very poor. The condition of the surface is an important factor. To some metallic surfaces rubber adheres more firmly when a slight chemical action occurs between rubber compound and metal during vulcanization.

In order to secure an indication of the tendency of rubber to adhere to filler particles, the following experiments were made: The surfaces of panes of glass were prepared in a variety of ways

[12] SCHIPPEL, *J. Ind. Eng. Chem.*, 12, 33 (1920).

[13] GREEN, *J. Ind. Eng. Chem.*, 13, 1029 (1921).

and heated in contact with sheets of a raw vulcanizable rubber compound composed as follows:

Washed smoked sheet	84.2
Sulphur	5.2
Zinc oxide	4.2
Litharge	4.2
Paraffin	2.2

The various preparations of the surfaces of the glass included washing alone, washing and drying *in vacuo*, coating with cotton-seed oil, paraffin, gutta percha or rubber solutions, heating in air or *vacuo* and cooling in vapors of benzine or ammonia. Most of the treatments resulted in an adherence of rubber to glass somewhat superior to the adherence to untreated glass, but in no case was there firm adhesion. Similar tests were carried out with various metals and also with corks and marble slabs, the specimens being used without any special preparation of the surfaces. It was apparent that in most cases the adhesion between rubber and foreign materials is very poor. Evidently when chemical action takes place, as in the case of brass and copper, adhesion may or may not occur, depending upon the adherence of the chemical product of reaction (sulphides in case of metals) to the metal.

Whether or not actual adhesion occurs between rubber and filler particles, an adsorbed gas film on the surface of filler particles might have an important influence on the volume of interfacial voids. It is obvious that the gas per unit area of surface rather than per unit weight of filler would determine cementation of filler and rubber magma. The area of surface is controlled by particle size. Dr. R. M. Burns has made a special study of the surface area and gas adsorbed by several fillers and supplied the following data. Measurements of rapidity of sedimentation were used to determine the distribution of particle size from which the surface per gram of filler was computed by application of Stokes' law on the assumption that the particles are spherical. Determinations were also made by pumping out at high vacuum at a temperature of 360° C. of the adsorbed gases per unit area of surface of the fillers which had been brought to equilibrium with atmospheric air. These are shown in Table VII. The precision of these adsorption measurements is about 5 per cent.

Experience with scores of rubber compounds involving the

fillers shown in Table VII seems to show a relationship between the amount of adsorbed gas per unit surface of fillers and the changes which rubber compounds containing the fillers show on prolonged immersion in water. The case of zinc oxide is especially interesting. Its particle size is very small and the surface per gram much higher than that of any other filler studied. Yet the gas adsorbed per unit area of surface is of a lower order than that of other fillers, and in spite of its fineness zinc oxide produces rubber compounds of maximum electrical stability on immersion in water. (Charts II and III.) Other factors, of course, may be responsible in whole or in part for this result.

TABLE VII.

Adsorption of Gas on Fillers.

Sample.	Surface. $m.^2/gm.$	Oxygen and Nitrogen. $c.c./m.^2$	Carbon Dioxide. $c.c./m.^2$	Total Gas. $c.c./m.^2$	Water, per Cent.
Fused silica	0.22	0.327	0.140	0.467	0.080
Fused silica, 1000°–1 hour	0.21	0.237	0.020	0.257	0.025
Fused silica, washed	0.22	0.232	0.179	0.411	0.073
Fused silica, 1000°–10 hours	0.21	0.144	0.003	0.147	0.009
"Silica Smoke," *400 mesh	0.65	0.080	0.252	0.332	0.544
"Silica Smoke," 200 mesh	0.20	0.180	0.147	0.327	0.153
"Silica Smoke," 1000°	0.20	0.212	0.004	0.216	0.014
Whiting number 26	0.12	0.175	0.222	0.397	0.043
Franch talc	0.22	0.142	....	....	....
Zinc oxide	3.46	0.002	0.010	0.012	0.140
Zinc oxide, sample 2	3.46	0.000	0.024	0.024	0.014

*A commercial grade of ground crystalline silica.

Among varieties of silica of equal fineness those which adsorb the least gas are those which produce the most stable compounds.

Evidence of this sort led to attempts to remove the gas from filler particles. A method which was extensively tried is to masticate the rubber compound under vacuum before forming into sheets or cores. This method produces material having substantially the same water absorption but of slightly greater electrical stability. It is not surprising that this should be the case, as it is impossible to bring the rubber mass to a temperature high enough to remove adsorbed gas thoroughly, and mastication of a tough material under a high vacuum is impracticable. Fortunately, a more efficient and simpler method is available for fused silica, *i.e.*, roasting at a high temperature, say 1000° C., for several hours. As shown in Table VII, 50 to 75 per cent. of the gas is removed by this means. Evidence was obtained by

X-ray examination that under these circumstances the surfaces of the fused silica particles are converted into a crystalline variety of silica, tridymite, which appears to have a diminished tendency to adsorb gas. Accordingly, the filler may be compounded with rubber by the usual means. Table VIII gives data illustrating the degree of advantage gained by this treatment in compounds otherwise identical.

The influence of gas on adhesion is of less importance in the case of organic fillers. Although the gas adsorbed on particles of hard rubber dust has not been studied, apparently adsorbed gases do not play an important rôle in the case of this filler, as

TABLE VIII.

Effect of Roasting Silica Filler.

Sample No.	Roasting.	Maximum Change on Immersion in Sea Water. Per Cent.		Total Gas Adsorbed on Filler, c.c./m.^{2}
		Water Content.	Dielectric Constant.	
233	1 hr. at 1000° C.	.51	7.4	.257
241	None	.60	16.3	.467

numerous attempts to improve its compounds with rubber by mastication in vacuum have not given the slightest evidence of benefit. Probably this filler is sufficiently plastic at the temperature of vulcanization so that any gases adsorbed at the surfaces of the original particle distribute themselves throughout the particle. If this takes place no interference with the wetting of the particle by the rubber magma would be expected.

Contamination of the surfaces of filler particles otherwise than by adsorbed gases may at times be important. Such evidence came to light in an attempt to separate fused silica into lots of varying fineness by means of an air separator. All the silica so separated, of whatever fineness, was found to produce rubber compounds which upon immersion in water absorbed more water and underwent several times greater electrical changes as compared with compounds made from the original material. A mixture of the various grades of material, according to fineness in the proportions in which they occurred in the original material, also produced inferior sheets. On heating samples of such air-separated silica under vacuum, there was evidence of the distillation of minute amounts of oil from the material. It was also

found that roasting of the silica at about 1000° C. largely, but not fully, remedied the difficulty. It seems possible that the particles of silica were electrified by the air stream and attracted from the fan not only droplets of lubricating oil of minute size, but also some fine dust which was not removed by roasting. The contaminations were, of course, too small to be detected by ordinary chemical means. The separator apparatus was scrupulously clean in the ordinary sense of the term, yet the contamination was sufficient to be ruinous.

It might be supposed that the method of working a filled rubber stock would be of great importance in securing good cementation of filler particles in the rubber magma. However, this does not appear to be the case, provided there is a reasonable degree of thoroughness of mixing. Within the limits of good commercial practice, variation in the length of time or temperature of milling is unimportant unless pronounced "scorching" of the stock occurs. "Scorching" is a term applied to describe an incipient vulcanization or toughening of rubber which is worked at too high a temperature. Even a moderate amount of "scorching" may occur without serious damage to the electrical stability of the final product. It is likewise of minor consequence whether a rubber stock is calendered hot or cold within the limits which permit meeting dimensional requirements. Apparently it is also true that a rubber compound, when extruded, has properties after vulcanization practically very similar to those of a calendered stock. This was tested by extruding sheets of silica and hard rubber dust compounds and comparing their electrical stability in water with calendered sheets. Apparently the effects of mechanical working are largely eliminated by subsequent vulcanization.[14]

CORRELATION OF ELECTRICAL CHANGES WITH WATER ABSORPTION.

As indicated in the various tables presented, the absorption of water by rubber is attended by an increase in dielectric constant. Table IX shows the changes in capacity and in water content taking place in a typical washed and a typical unwashed, unfilled, soft, vulcanized rubber in the form of sheets .050 inch thick, immersed in 3.5 per cent. salt solution.

A similar rough relationship between water absorption and

[14] WIEGAND, *India Rubber J.*, 64, 815 (1922).

capacity change is observable also in filled rubbers. In general the leakance also rises with water content and the insulation resistance declines. These changes in ordinary commercial rubbers are often extremely large, but in material properly treated to reduce water adsorption are comparable with those occurring in gutta percha.

Although there is a rough parallelism between water content and electrical characteristics, there is no exact mathematical relationship as shown by evidence presented in the tables and charts.

TABLE IX.

Changes in Water Content and Capacity of Unfilled Vulcanized Rubber.

Unwashed Smoked Sheet Vulcanized Rubber. Sample 27-B.			Washed Smoked Sheet Vulcanized Rubber. Sample 577-D-1.		
Immersion Period, Days.	Water Content, per Cent.	Capacity Change, per Cent.	Immersion Period, Days.	Water Content, per Cent.	Capacity Change, per Cent.
7	1.74	12.7	7	0.65	8.0
14	2.34	13.7	16	.71	8.0
30	3.15	15.8	29	.77	7.5
60	4.03	20.2	91	.89	8.0
150	5.40	22.9	197	0.95	7.0
190	5.79	25.2	....	...	...

The increase of dielectric constant, for example, is not strictly proportional to the amount of water absorbed by rubber whether filled or unfilled. In all cases the effect of the first increments of water is larger than that of the later increments. Indeed the effect of the earlier increments is often of a higher order than that which would be predicted on the basis of any well-founded law of mixtures such as the Lorenz-Lorentz formula.[15] This suggests two modes of distribution of the water through the rubber magma. The larger effect of the earlier increments may well be due to a non-homogeneous distribution probably somewhat in pores and to a much greater extent at the surfaces of filler particles. If a filler particle were completely surrounded by a conducting aqueous solution which is in turn enclosed in a rubber envelope, and the conductivity of the aqueous solution were high, the capacity would approach that which would result if the filler particle and aqueous solution were replaced by a conducting particle. It is only on some such basis as this that we can account for the very high apparent dielectric constant of a rubber com-

[15] *Ann. d. Phys.*, (3), 9, 641 (1880).

pound such as Sample 216 (Table V) when wet. The following examples illustrate our evidence that a non-homogeneous distribution of water is productive of high capacity. Sample 46-B was made of barytes containing very little soluble matter. Yet after seven months in sea water the dielectric constant increased from 4.85 to 16.0, or 230 per cent., while only 0.8 per cent. water adsorption occurred. The ratio of the two increases was $\frac{230}{0.8} = 288$. By contrast, sample 50-B, an unfilled vulcanized rubber showed a capacity gain of 17 per cent., a water gain of 4 per cent. and a ratio of the two increases of only 4.2. Many other like examples could be cited. Chart III shows in part how a variety of rubber compounds behave electrically on immersion in sea water.

However, in general, the influence of the factors controlling the dielectric constants of rubber compounds have been evaluated for practical purposes in a fairly consistent though far from mathematical manner. This is less true of the factors controlling phase angle. Fortunately rubber compounds which exhibit a satisfactory degree of stability of dielectric constant are usually satisfactory as to phase angle as far as our experience extends. The latter electrical property is subject to wide and often unaccountable fluctuations in many rubber compounds. Enormous changes have in a number of instances been associated with considerable amounts of water soluble matter in fillers (Table V). Numerous anomalies which we have encountered indicate that the phase angle of rubber compounds, as well as other insulating materials, requires a great deal more study.

SUMMARY.

(1) Soft vulcanized rubber, though not well adapted to some of the processes of manufacture of submarine cable, can be so made as to be mechanically and electrically suitable and to withstand the action of sea water in a manner comparable with that of gutta percha over a period of a few years. Whether such rubber will retain these characteristics for decades remains to be demonstrated, but it seems probable that it will.

(2) The principal factor to be controlled in producing this result is the amount of water absorbed by the rubber.

(3) Osmotic pressure of internal and external fluids is of

prime importance in governing the in-flow of water into rubber and gutta percha.

(4) Lowered water absorption is achieved by removal of water-soluble matter from the rubber, the choice of an insoluble, non-reactive filler of suitable particle size and having a minimum of adsorbed gases or other contamination on its surfaces.

(5) The electrical characteristics of rubber compounds and of gutta percha are clearly related to their water content but are not simple functions of the water content.

(6) It appears that the mode of distribution of water is also extremely important.

(7) Most fillers for rubber compounds are not suitable for submarine insulation, either because of undesirable intrinsic electrical properties or because they are conducive to changes incident to water absorption. Hard rubber dust, silica and zinc oxide are the best fillers from these standpoints so far as known.

The Latent Heat of Fusion of Some Metals. J. H. AWBERY and EZER GRIFFITHS. (*Proc. Phys. Soc. London*, Aug. 15, 1926.)—It is surprising to learn that the latent heat of fusion of aluminum varies according to the experimenter from 64 to 94 calories per gram and that these divergent results are not the product of the earlier days of the nineteenth century, but of 1914 and 1919, respectively. In the cases of antimony and magnesium the authors state that they were unable to find any recorded values. They preface their paper by a valuable appraisal of the work of their predecessors from Person to Walter P. White.

For their purpose they selected the method of mixtures wherein the temperature rise of a mass of water is measured upon the introduction of a heated body. This method is beset by two main sources of error. In the first place there is a loss of heat during the transfer of the metal from the furnace to the water. To reduce the importance of this loss larger quantities of metal were used. If the dimensions of the metallic mass be doubled, its capacity for heat becomes eight times as great, while its surface and consequently its heat loss in transit is only four times as great. Thus four pounds of aluminum and seven pounds of lead were employed. Furthermore the metal, whether solid or liquid, was always transferred to the calorimeter while contained in a crucible. The empty crucible, heated to 500° C., was found by a thermo-element to lose 2.2° during transfer. As its heat capacity was about one-tenth of that of itself with the addition of the metal, it follows that in the latter condition its drop of temperature would be about .2°, the assumption being that the same number of heat units would be lost by the full as by the empty crucible.

The second objection is that steam is formed when the hot metal touches the water. This was met by lowering the crucible with its charge of metal into a vessel containing no water but surrounded by the water of the calorimeter. The lid of the instrument was then closed and the vessel was pulled down under the surface of the water. In the case of magnesium, which reacts with water, the metal had to be kept throughout in a metal container with soldered joints.

The heat of fusion was calculated by subtracting the heat given up by a gram of the solid metal at the melting-point in falling to 20° from the heat emitted by a gram of the liquid metal in descending from the melting-point through the same temperature range.

The mean specific heat of aluminum, a by-product of the experiment, is found to be .66 between 657° and 760°.

These are the results obtained:

Metal.	Melting-point. °C.	Latent Heat. Calories per gram.
Aluminum	657	92.4
Antimony	630	24.3
Bismuth	269	13.0
Lead	327	6.2
Magnesium	644	46.5
Tin	232	14.6
Zinc	420	26.6

G. F. S.

More Weather Studies Help Aviation, Fire Fighting and Lake Level Work.—Among the outstanding features of the work of the Weather Bureau during the past fiscal year, as outlined by C. F. Marvin, Chief, is mentioned the beginning of the extension of the activities of the bureau in support of aviation. Increased appropriations available in the present year will make it possible to carry out the plans formulated in coöperation with the military branches of the government.

Certain special appropriations have also been released with which to conduct fire weather warning work. The program of meteorological service in aid of forest fire fighting is a coöperative one in which the forest fire fighting agencies and forecasters and meteorologists who supply weather information are engaged.

An important piece of work completed by the bureau is the publication of a carefully prepared digest of all available precipitation data on the Great Lakes region from 1875 to 1924. For a score of years or so there has been a deficiency of precipitation in this region and if a period of increased precipitation occurs in the future it will mean an increase in lake levels. Changes in lake level during recent years have been regarded with anxiety, and it is hoped that the information gathered will help to stabilize thought regarding the causes of changes of lake level.

A CONTINUOUS INTEGRAPH.*

BY

V. BUSH, F. D. GAGE, and H. R. STEWART.

Electrical Engineering Department, Massachusetts Institute of Technology.

ABSTRACT.

A MECHANICAL integraph has been developed which plots continuously the integral of the product of two functions. It uses the principle of the electrical integrating watthour-meter combined with a moving table. Errors of the machine have been reduced to an average of 1 per cent. for common uses. By cross-connecting the device in a simple mechanical way, it is possible to solve certain types of integral equations. A link motion has been added which plots the product of two given functions. Various uses have been made of the instrument for solving problems in connection with electrical circuits, continuous beams, etc., and certain problems involving integral equations.

PURPOSE.

Engineers and physicists are depending increasingly upon mechanical devices as aids in computation. This tendency is bound to grow as analysis is extended to problems of greater inherent complexity.

Business office practice has been revolutionized by the advent of computing machines. These deal almost entirely in terms of numbers, as indeed does the business man, and machines are available which will add, multiply, tabulate, and perform other useful operations on sets of figures introduced into the machines by pressing keys. Applied physics, and in fact many other branches of science, frequently deal, however, with functions as a whole, and usually resort to figures only as a rather laborious means of dealing with functions or the curves which represent them.

Accordingly there is need for computing machines which will deal directly with the functions themselves; machines, in other words, into which may be introduced graphically the scientific relationships with which we deal, and which will perform useful operations upon them directly as curves.

* Communicated by the Authors.

One such operation is multiplication, and another and more important one is integration. The machine here presented is a multiplier and integraph which operates continuously, rather than point by point. It is only a beginning, of course, and various similar machines are necessary. Still it is thought to emphasize and facilitate rather more than has been done heretofore a type of functional computation which has large possibilities.

Mechanical integrators usually evaluate the definite integral between given fixed limits. There has been a need for a machine which would continuously evaluate and plot the integral as a function of a variable upper limit. There are many physical problems involving computations of this sort, especially where the quantity under the sign is the product of two known functions. The present machine, which we have called an integraph, since it records the result of an integration in the form of a plot or graph, has therefore been developed to evaluate $F(x)$ against x from the expression

$$F(x) = \int_a^x f_1(x) f_2(x) dx$$

where f_1 and f_2 are known functions, formal or empirical. An auxiliary device plots the product of the known functions rather than the integral of the product.

An interesting adaptation of the device has developed during its use. It has been found that by an expedient fairly well described as "back-coupling" it may be used to solve certain types of integral equations. As an example the equation

$$\phi(x) = \int_a^x f(x)\, \phi(x)\, dx$$

may be solved for an unknown ϕ when f is known. The integral equation method of attack on physical problems is so powerful, and so evidently limited in its power by the difficulty of solving even simple integral equations, that this result suggests very attractive possibilities. This use has been extended to other equations than the above, although only to a limited general type. The whole question deserves rather more extended treatment than can be given herein, but the adaptation to simple types of equations will be treated, after the construction of the machine has been described.

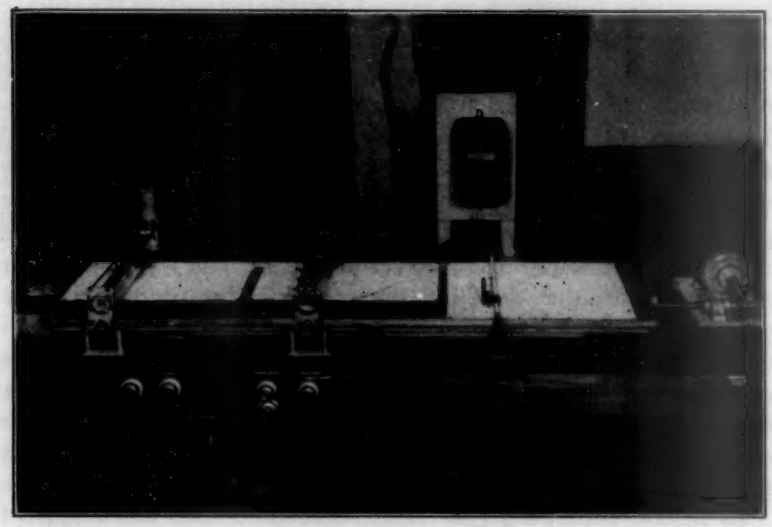
DESCRIPTION OF THE APPARATUS.

The two functions to be multiplied together under the integral sign are plotted to convenient scale on cross-section paper and fastened to a table which moves in the direction of their x-axes at constant speed. The right-hand plot has its x-axis as an extension of the x-axis of the left-hand plot. Located across the table at the same distance apart as the y-axes of the two curves are two slide-wire potentiometers carrying constant current, the sliders of which are furnished with pointers by which the operators follow the curves fastened to the moving table. The midpoints of the potentiometer resistances are tapped, thereby permitting the voltage drop from the midpoint to the slider contact to be obtained in each case. This voltage drop is therefore proportional to the ordinate of the plotted curve when the slider pointer rests on the curve. These two voltage drops are applied respectively to the potential and current coils of a modified Thomson direct-current integrating watthour-meter. We thereby secure the means of integrating the product of the two plotted functions when the table moves, and the plots are followed by the slider pointers. The watthour-meter may rotate in either direction since the function product may be positive or negative. The reading of the watthour-meter is recorded continuously by a relay arrangement as the integration is performed. This relay system is actuated by the watthour-meter by means of substantially frictionless contacts, and controls the operation of a small reversible servo-motor in such a manner that the motor follows exactly the rotation of the watthour-meter. This motor, through a gear train, operates a lead screw located across the table at a convenient distance from the slide wires. A pencil, attached to a split nut which rests on the lead screw, is thus driven by the motor and draws, on blank paper fastened to the table, the result. The instantaneous speed of the watthour-meter, and hence the speed of the recording pencil across the table, is thus made at each instant equal to the product of the corresponding ordinates of the two plotted functions. Since the table moves at constant speed in a direction perpendicular to the motion of the pencil, the instantaneous slope of the line drawn by the pencil will be proportional to this product, and the curve drawn is therefore the integral of the product of the two plotted functions. The arrangement of the parts is shown in Fig. 1.

The control switches are shown at the lower left of the photo-

graph, and all wiring is under the table. At the extreme right of the picture is the synchronous driving motor and its gears which operate the moving table. The table has three sheets of paper on it. On the left-hand sheet and the centre sheet are plotted the known functions to be multiplied and integrated. The right-hand sheet is the one upon which the solution is drawn. A slide wire is located above each of two left-hand sheets. The pencil and its driving mechanism are over the right-hand sheet. To the rear of the pencil-driving mechanism is located the small servo-motor

FIG. 1.



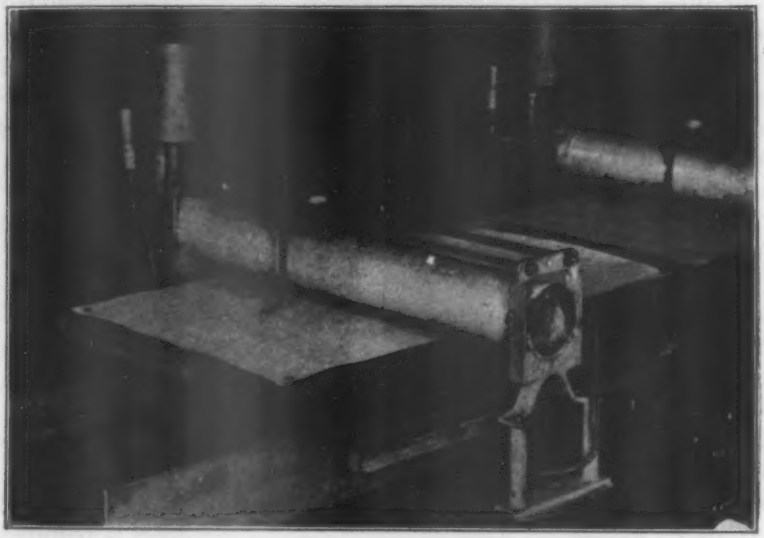
General view of integraph.

which follows the motion of the watthour-meter above it. The relays are encased in glass and situated to the left of the small motor.

The moving table is six feet long and eighteen inches wide, and is reinforced by a piece of quarter-inch cold-rolled steel of the same dimensions. On the bottom of this steel plate are grooved steel blocks, which serve as bearing surfaces and slide on top of ways made of a piece of channel iron. The synchronous driving motor is connected to the moving table through reduction gears and a long lead screw. The coupling between the table and the lead screw is by means of a half nut, mounted on the end of the table and so arranged that it can be raised and disengaged from

the lead screw in order that the table may be moved back and forth at will without running the motor. A constant speed for the table motion is secured by using a synchronous motor of small power supplied from a constant frequency source. The normal speed of the table is chosen to be one-half foot per minute, and one-half this speed can be obtained. The accuracy of the result obtained depends largely upon the accuracy of the operators in following the graphs as they move under the sliders, and the slower speed is used to obtain accuracy when abrupt changes

FIG. 2.



The potentiometers.

occur in the plotted functions. With the size of moving table used it is possible to allow a travel of about twenty-four inches for the graphs. An automatic stop is provided to cut off the driving motor at the end of the allowed travel.

The slide-wire potentiometers are of the tubular type and sufficiently long to allow them to straddle the moving table.

Fig. 2 shows the method of mounting these slide wires by means of small tubular pillars. There are five hundred and eighty turns of No. 20 B. and S. gage enameled constantan wire on each slide wire. There are consequently two hundred and ninety steps of voltage between the centre or neutral taps on the slide wires

and the two extremes of voltage on their outer edges. To prevent shifting of wires the whole winding is heavily enameled with an ordinary white enamel. In order that the currents taken from the slide wires to the watthour-meter may be small in comparison with the currents which flow through them, the resistance of the slide wires is only 100 ohms, allowing a current of two and three-tenths amperes to flow from the 230-volt storage battery source through each slide wire. A constant temperature of 100° C. of the slide wires is secured. Boiling water is kept in the slide wires by means of plugs which are soldered in each end of the tubes. The small cylindrical tank at the left of the picture and mounted on the slide wire is a condenser to restore the vaporized water to the rheostat. A glass tube on the far end acts as a gage. Instead of the usual triangular rod used for mounting the sliders on the slide wires, two rods are used to give greater rigidity. Brass blocks with proper-sized holes for the rods are used for the sliders, and to these are fastened small square brass rods bent down over the graph sheets and used as holders for the pointers. The pointers are made up of small pins weighted at the tops and slide through guide holes in the holders. In the photograph one pin is located at the centre of the graph on the moving table. For accurate operation of the slider there is a rack and pinion located on one of the cross-rods. The pinion is attached to the slider by a sleeve bearing and has a hand knob on the end of its shaft capable of rotating the pinion. The rack is mounted permanently on the slide-wire framework and acts as one of the slider supports. This reduction gear is necessary for accuracy of control on the operator's part, although when the motion of the slider is required to be more rapid than is possible by use of the rack and pinion, it can be moved directly and without using the latter. The sliding electrical contacts between the sliders and the slide wires are spring-actuated steel pins.

The watthour-meter used is a 10-ampere, 110-volt instrument of standard design, made by the General Electric Company, on which, however, several modifications have been made. In the first place, the meter is required to run backward, and this forces the removal of the light load coil usually used to offset static friction. The second requirement is that the two windings shall take a small current, and this is taken care of by rewinding the current coil with about eighteen thousand turns of small wire

capable of carrying forty milliamperes. This winding gives the same number of maximum ampere-turns for the field as the original high-current winding. The potential circuit of the meter was left unchanged except that the light load coil was replaced by an external resistance. One of the two drag magnets was

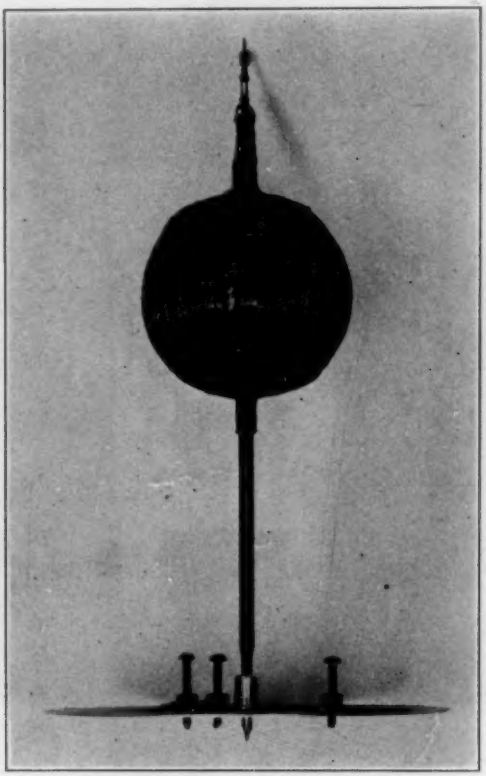
FIG. 3.

The watt-hourmeter and servo mechanism.

removed from the frame. This speeded up the rotation of the disc of the meter and, within the range of speeds found necessary for most curves, increased the percentage accuracy of the readings of the machine. The commutator friction of the meter armature was reduced by reducing the brush pressure. Only about 3 per cent. to 5 per cent. of the friction of the meter was found to occur at the commutator, the balance residing at the pivot which

carries the weight of the moving element. Another change was the use of an oscillating support bearing for the armature pivot. The jewel of the meter, which is ordinarily mounted on the frame, is now mounted at the centre of a small rotatable table which, as will be seen later, is in continuous oscillation and which follows

FIG. 4.



The disc and contacts.

the armature of the meter in its movements. This change practically eliminates the effect of friction at this point, and makes the watthour-meter really a precision instrument. Before this change, the watthour-meter would actually stop when the current in each coil was reduced to 5 per cent. of rated value. As a result of the change, it operates at this same reduced load with an error of only about 1 per cent.

The relays and servo-motor mechanism are shown in Fig. 3. Avoidance of friction in operating the relay circuit required a special form of contact on the watthour-meter disc, which is shown in Fig. 4. Three machine screws are carried by the disc, and the lower tips of these make the connections which operate the relays. The screws are platinum-tipped, and the lower surface of the platinum is wet with a drop of mercury which forms a meniscus contact surface. The other half of the contact system is mounted on a small round bakelite table about three inches in diameter. This small table rotates in a sleeve bearing with its upper surface about one-quarter of an inch below the watthour-meter disc, and it revolves about the same centre line as the disc. On its upper flat surface are strips of metal for the lower contacts, and these contacts are touched by the mercury drops carried by the screws. It is this bakelite table which also supports the jeweled bearing for the armature pivot, as described above.

The inside screw of the meter disc sweeps around the surface of the bakelite table top in a circle, and its mercury drop makes contact with a circular nickel-plated strip of metal. This contact is the ground contact and makes connection at all times. The screw with the largest radius is connected in the circuit of a two-way relay through its contact with either one of two semi-circular nickel-plated strips embedded in the surface of the bakelite table. The remaining screw makes contact with a strip which is shaped like an incomplete circle. This operates a single-contact relay which we shall call the high-speed relay. The metal strips on the moving bakelite table are connected with four brushes on the under side by means of slip rings. The bakelite table is rotated by means of a gear train from the servo-motor, which in turn is controlled by the relay system. It has a constant field and its armature is supplied through the two-contact relay with 115 volts from either side of the 230-volt source to enable it to run in either direction. A resistance is placed in series with the armature of the servo-motor to reduce the speed and the abruptness of starting. When the high-speed relay closes, this resistance is about four-fifths cut out in order to increase the speed of the motor in either direction. Fig. 3 shows the watthour-meter with the cover removed, the gear train and servo-motor being located below the meter disc. The large circular piece of brass in front of the meter armature is the front of the spool for the rewound current coil.

One of the relays may be seen at the lower left. The shaft at the extreme lower right is the lead screw which drives the recording pencil. It is connected to the servo-motor by direct gearing as shown. The operation of this part of the integraph is rather complicated, but has proved to be quite reliable. Assume the watthour-meter to be rotating slowly. The outermost screw will have its mercury tip drawn over one of the semi-circular conductors embedded in the bakelite table. This closes the circuit of one side of the two-way relay, which acts to start the servo-motor in such a direction as to follow the meter. Since it is unlikely that the meter speed will exactly equal the bakelite table speed, we will assume the meter to lag behind the table and cause the mercury contactor to slide off the semi-circular contactor into the neutral zone between the contacts. The relay is, however, not stable in midposition so it will not as yet open. Hence, the mercury contact slides still farther back and onto the other semi-circular strip which operates the other coil of the relay to cause a reversal of the servo-motor and the bakelite table. If the meter is going at such a speed as to make this process continuous, the servo-motor and all of the gearing attached is made to oscillate back and forth at a frequency of about two oscillations per second. If the meter disc turns faster than the bakelite table at its normal low speed, the third contactor comes into play and makes connections which increase the speed of the servo-motor to such a speed as will be faster than that ever required by the meter disc. It is thus possible to have the mechanism oscillate back and forth between the low and high speeds. Since it is rare to have the meter turning at the normal speed of the bakelite table, the motor and gear train are practically always in a state of oscillation, following faithfully, however, the movements of the meter, whatever they may be.

This oscillation provided the method of cutting down friction of the meter. The jeweled bearing which is normally mounted on the frame of the watthour-meter was removed and mounted on the centre of the bakelite table. Since the motion is always present, static friction in the bearing is avoided and a great increase in sensitivity is realized. The applied torque to the meter does not need to overcome a static friction in order to produce a motion, but simply adds and subtracts alternately from the dynamic friction torque at the pivot as its jewel support oscillates.

Thus, while there is no appreciable net drift of the meter due to the oscillation when either coil current is zero, the slightest torque will produce a proportionate speed of rotation.

Since the lead screw which operates the recording pencil is directly connected to the servo-motor, it shares the oscillatory motion of this system, and the resultant record is a slightly wavy line. A loose gear coupling to the lead screw was found to improve the smoothness of the record. The wavy line is no great disadvantage, as the final curve may be drawn as a smooth line through the middle of the ripples in the record.

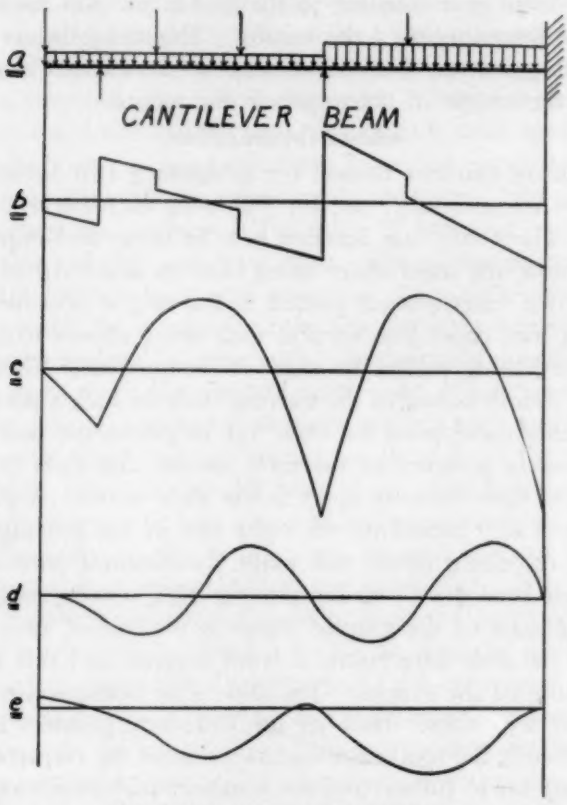
SIMPLE INTEGRATIONS.

When the machine is used for integrating two functions, two operators are necessary, one for following each curve. The special case where only one function is to be integrated requires only one operator, the other slider being then set at a fixed value.

The two functions are plotted to the largest possible scale on 18 by 24 inch paper, the vertical scale being chosen to insure the final record being within the scope of the machine. These graphs are then thumb-tacked to the moving table in such a position that their x-axes correspond for their full length to the zero point of the slide-wire pointers as the table moves, and their y-axes are spaced the same distance apart as the slide wires. A plain sheet of paper is also tacked on the right side of the moving table so that the recording pencil will make its complete path across it. A line has been drawn on the moving table corresponding to the horizontal axis of the graphs, which is the line of zero displacement of the slide-wire pointers from neutral, and this facilitates the locating of the graphs. The slide-wire voltages are adjusted to exactly 230 volts. Both of the slide-wire pointers are set at approximately the root-mean-square value of the respective curves which they are to follow, and the watthour-meter coils are allowed to heat up to the temperature corresponding. This requires about thirty minutes. This precaution eliminates the error due to temperature variation of the watthour-meter coils to a sufficient extent so that the limit of accuracy is then practically the error caused by failure of the operators exactly to follow the plotted curves. Of course, this warming-up period is necessary only on precise work and may be much shortened when the instrument is being used continuously. In the meantime the moving table is

pushed to its extreme right-hand position and engaged with its lead screw by means of the half nut at the right-hand side of the table. The driving motor of the moving table is turned on until it has moved the table to the left a distance which brings the two slide-wire pointers to the vertical axis of the plots of the two

FIG. 5.



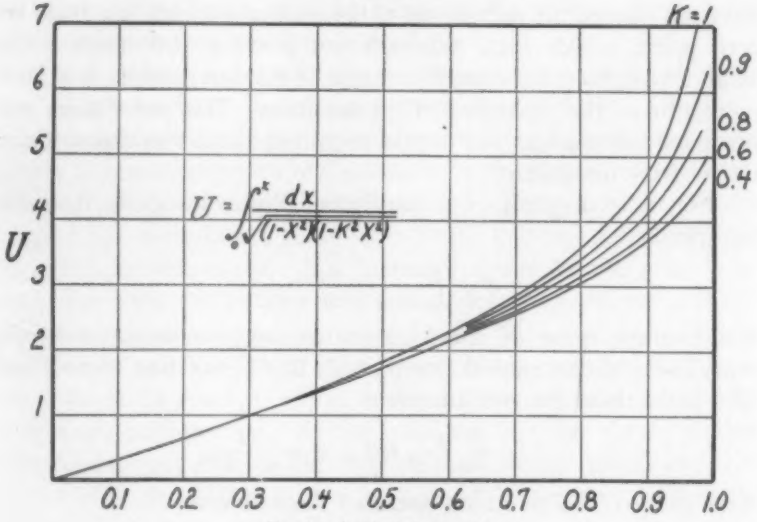
Solution of a continuous beam problem.

functions. The gear ratio between the servo-motor and the pencil lead screw is adjusted so that the pencil will record a maximum amplitude of displacement without going off the record sheet. A fair estimate of the gear ratio can usually be made by inspection of the graphs to be multiplied and integrated. It is sometimes possible to allow the pencil a greater amplitude of motion by locating the zero axis of the result graph toward the bottom

or top of the sheet, that is by starting the pencil at this point and increasing the gear ratio between the motor and the pencil.

A starting switch is provided which simultaneously controls the watthour-meter circuits and the table motor. This is now turned off, the pointers are moved to the starting-points on the graphs, and the relay and servo-motor switches are turned on. The table and the watthour-meter element are now motionless, but the relays are working and the servo-motor is oscillating about

FIG. 6.



Family of elliptic integrals.

a zero position, causing the pencil to oscillate in turn through an amplitude of about 0.2 mm.

The starting switch is now closed, and the table motor and watthour-meter go into action simultaneously.

Each operator traces carefully the curve under his supervision. Four minutes is the time of the run at normal speed. After each few curves taken, a calibration run should be made to insure freedom from error. This is done by setting the pointers of each of the slide wires on a convenient number of units on each graph and making a run as before. The slope of the straight line thus obtained gives the calibration directly.

Two examples of results obtained by plain integration are shown in Figs. 5 and 6.

Fig. 5 shows the results of three consecutive integrations in the solution of a loaded continuous beam problem. The upper figure (a) is a sketch of the beam shown with its supports and loads. Next below is the graph of the shear which is plotted for use on the integraph. The first integration result is shown in part (c) of the figure and represents the bending moment curve. The next and second integration is part (d), and this represents the slope curve. The final curve, being the integral of the slope curve, is the deflection curve. The errors of three integrations have not caused the deflections at the supports to fall far from the zero point, which fact, although not proving the errors to be small throughout the complete range of the integration, is a good indication of the capability of the machine. This curve sheet was made by one operator, of course requiring three successive operations of the integraph.

Fig. 6 is a graph of a family of elliptic integrals from the equation

$$U = \int_0^x \frac{dx}{(1 - x^2)(1 - k^2x^2)}$$

The average error of these curves was approximately 0.85 per cent. in the actual record from which this figure was transcribed. To obtain them the two functions

$$f_1(x) = \frac{1}{1 - x^2} \text{ and } f_2(x) = \frac{1}{1 - k^2x^2}$$

were plotted, and two men operated the machine.

REDUCTION OF ERRORS.

The reduction of the slide-wire errors was accomplished in three ways: First, the use of small steps of voltage between turns; second, the maintenance of a uniform temperature at all points on the wire; and third, the use of a large current in the wire compared with that taken from it. The possible error due to change in voltage between turns is about one-third of 1 per cent. of the maximum reading, and correspondingly greater as the displacement of the pointers from the zero axis becomes smaller. This is an error which balances out to a great degree since the chances are that the next higher voltage will be used the same amount of time as the next lower voltage with reference to the true reading on the graph sheet. The maintenance of uniform temperature along the wire is accomplished by the water-cooling system. The

water is kept high in the tubes, and since the power consumption is fairly large, the heating of the slide wires keeps the water at boiling temperature. The water cooling also allows the use of smaller wire for the same current rating than would otherwise be permissible. The currents taken by the watthour-meter coils are each about 40 milliamperes. This requires the use of the currents of 2.3 amperes in the slide wires in order that the potentiometer error may not exceed 1 per cent. The maximum of this error occurs when the sliders are at the greatest possible distance from the zero potential taps, and the effect of the error is to give the meter coils voltages less than desired.

The watthour-meter contains two innovations for the elimination of errors, both having been mentioned before. The pivot friction error which, in the original meter, is so bothersome at low speeds, is almost completely eliminated by placing the meter armature shaft pivot in an oscillating bearing. The commutator brush friction has been found to be very small, and this effect opposes the potentiometer error. The mercury drops of the relay contacts slide over the surface of the nickeled strips on the bakelite table beneath with no perceptible friction. The fact that there is copper wire in the rewound current coil causes some heating error, especially if the machine is run without first warming up the coils to a mean temperature. If the coils are left for the half-hour with the current flowing through them as before mentioned, the error due to heating is about one-half of 1 per cent. on a curve of reasonable form.

There must be added to the above errors the human errors involved. The first and smallest error is the one in drawing the two graphs of the functions to be integrated. The second error is the one involved in the operators' failure to follow the curves accurately. This error may be as much as 2 or 3 per cent. for any given instant on the graph, but, where the operator overshoots once, he usually undershoots in return, and the highest probable error again lies within the 1 per cent. mark with a skilled operator. The third human error is in the drawing of a smooth ink curve through the mean value of the pencil result which, as has been pointed out, contains small back-and-forth oscillations. An over-all accuracy within 1 per cent. can be attained, and this is ample for most engineering work.

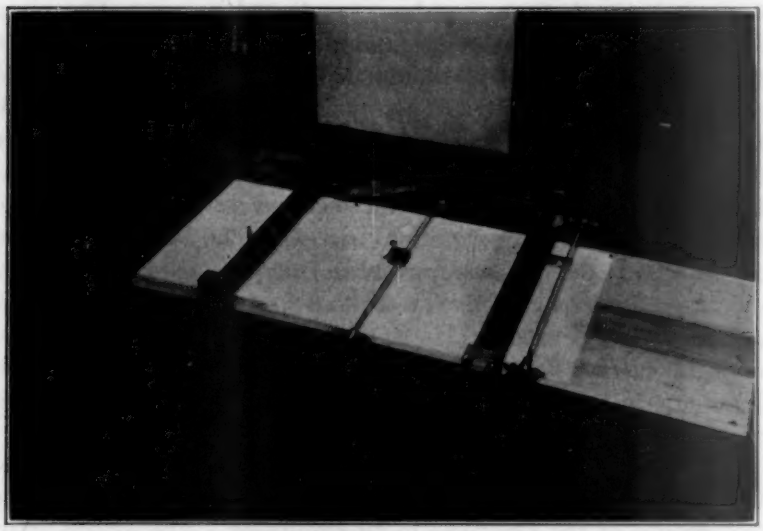
THE MULTIPLYING ATTACHMENT.

A link motion has been added by which two functions may be multiplied together, or the results of an integration multiplied by a function outside the sign. This multiplier utilizes an extension of the moving table of the integraph. Fig. 7 is a photograph of the multiplier, and Fig. 8 a diagram showing the link motion involved.

As the principle of this device is much the same as has been used in other link motions, the description will be made brief.

An arm (E) rotatable about a pivot at (J) carries a pin (X). The angle through which the arm is rotated is controlled by one

FIG. 7.



The multiplying mechanism.

ordinate to be multiplied, and the distance of the pin from the centre of the arm by the other ordinate to be multiplied. This pin engages in a slot (G) and moves a sliding rod (H) which carries the recording pencil (Q). The angular rotation of the arm is controlled by a yoke at (K) which engages with a fixed pin on the lower side of arm (E). It is evident that the displacement of the pencil will be proportional to the displacement of (K) and also to the distance of pin (X) from the centre of the arm. Thus the pencil records the product of these two displacements,

and if they are made equal to the ordinates of two curves on the moving table, the pencil will record continuously the product of the ordinates.

The displacement of (K) is made proportional to one ordinate by moving a pointer (P) on slider (C) so as to follow one curve, the motion being obtained through a crank (A) and lead screw (B). The other motion is not quite as simple. A lead screw (S) carries a pointer (O) which follows the second curve.

FIG. 8.

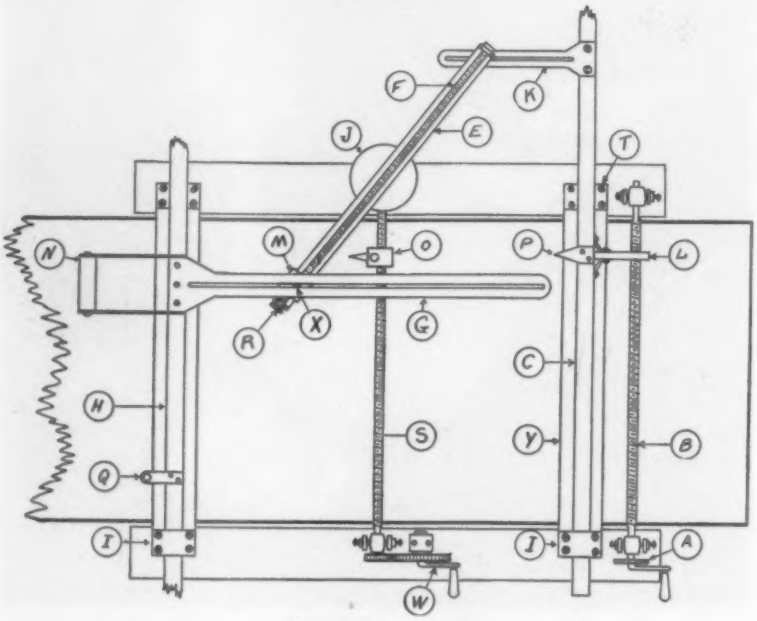


Diagram of multiplier.

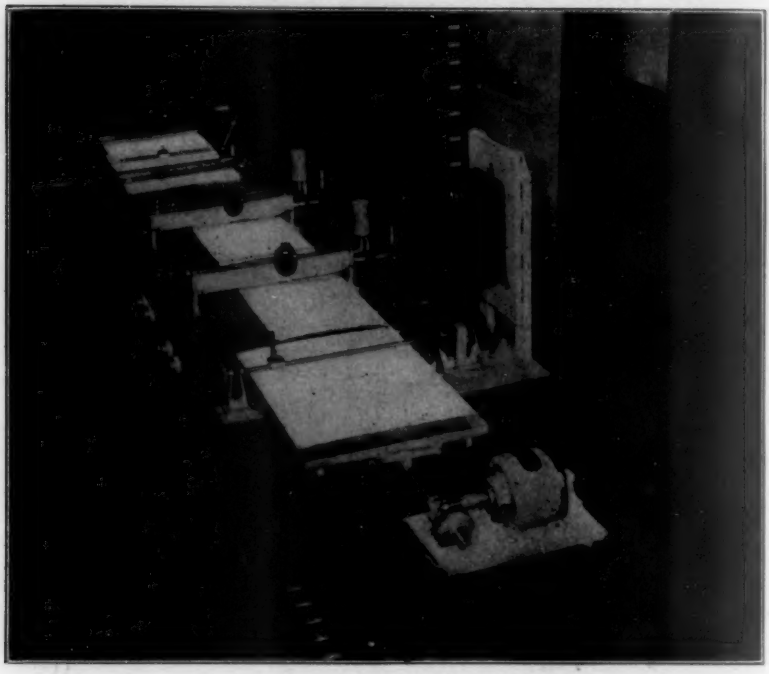
A train of gears is so arranged that as (S) is turned by crank (W) a second lead screw (F) on the arm (E) is turned an equal amount, and this latter carries the pin (X) by means of a slide and nut (M). This gear train operates along the axis of (J), the support of arm (E), and hence functions independently of the arm angle. Thus the displacement of pointer (O) from the centre line, and of pin (X) from the centre of the arm (E) are at all times equal.

By following ordinates with pointers (O) and (P) the attachment may thus be made to multiply two curves together, to re-

ciprocate a curve prior to integration, etc., and record the result by pencil (Q).

The integraph and multiplier may be interconnected, as is shown by the sprockets and chain shown in Fig. 9. When thus arranged one pointer (P) of the multiplier is caused to follow exactly the motion of the recording pencil of the integraph. If

FIG. 9.



The integraph and multiplier interconnected.

the other multiplier pointer is then made to follow the curve of a given function, the recording pencil of the multiplier traces the result of an integration multiplied by this given function. In other words, we will obtain from the device an evaluation of

$$f_3(x)\int_a^x f_1(x)f_2(x)\,dx$$

where f_1, f_2, and f_3 are given functions.

This arrangement has proved useful, as an example, for evaluating integrals of the form

$$E\omega \cos(\omega t + \theta) \int_0^t \cos \omega\lambda A(\lambda)\, d\lambda$$

which occur in the problem of the transients in circuits due to an applied alternating voltage

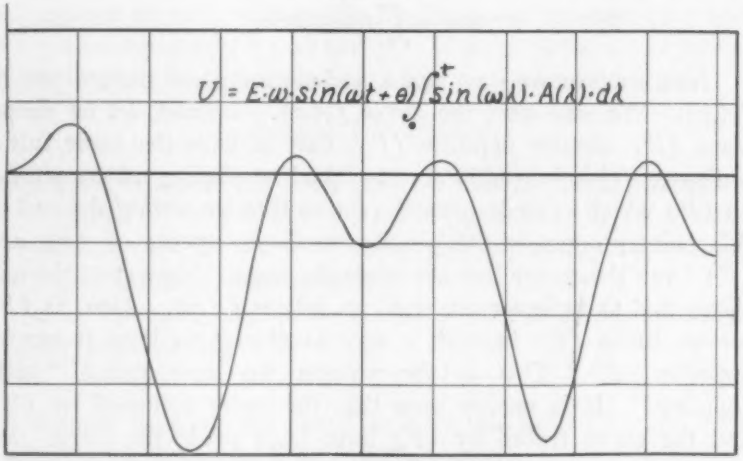
$$E \sin(\omega t + \theta)$$

where the behavior of the same circuit under unit constant applied voltage is given by

$$i = A(t)$$ [1]

The saving in labor is very large, for while it is often relatively

FIG. 10.



A continuous integraph.

easy to solve for the direct current transient in a network, the corresponding alternating-current transient is obtained only by much computation unless mechanical integrations may be readily performed in a continuous manner.

Fig. 10 gives a result of this sort obtained for the alternating-current transient on a smooth transmission line. Three curves, of which this is one, together make up the complete curve of entering current to the line. In this case $A(t)$, the direct-current transient for the line, is a discontinuous curve so that the benefits derived from graphical processes are especially noteworthy.

[1] J. R. CARSON, "Transient Oscillations of Networks," *Trans. A.I.E.E.*, 1919, Part I, p. 345.

SOLUTION OF INTEGRAL EQUATIONS.

Certain types of integral equations may be solved on the integraph, and some of these are of such form that their formal solution is obtainable only in the form of a series of integrals evaluated with difficulty. A direct solution of such equations mechanically in a single operation promises to be the most valuable feature of the device.

Suppose we designate the two pointers of the integraph (A) and (B), and the recording pencil (P). Then, if (A) and (B) are caused to follow two given functions $f_1(x)$ and $f_2(x)$, the pencil (P) will trace the result $\phi(x)$ of the integration.

$$\phi(x) = \int_a^x f_1(x) f_2(x)\, dx$$

Now suppose we start again, and place (A) on the ordinate of $f_1(x)$. We will omit the curve $f_2(x)$. Instead, let us simply place (B) directly opposite (P), that is with the same initial ordinate. This is readily accomplished by placing a long pointer on (B) which extends toward (P) so that by setting the end of this pointer opposite the pencil, the displacements of (B) and (P) from the centre line are made the same. Now start the machine, and as before cause (A) to follow $f_1(x)$. Also, as (P) moves, move (B) in such a way as always to keep it exactly opposite (P). This is the process we have called "back-coupling." It is readily seen that the curve followed by (B), and the curve traced by (P) have been made the same. Let us call this curve $\psi(x)$. We have caused the machine to solve the equation

$$\psi(x) = \int_a^x f_1(x)\, \psi(x)\, dx$$

where $f_1(x)$ is a known function, and $\psi(x)$ the unknown to be solved for. There will be a family of solutions corresponding to different values of a, that is, to different initial positions of the recording pencil.

An extension of this scheme may be readily made. Instead of holding (B) exactly opposite (P), let us hold it always at a fixed displacement (K) with respect to (P). This may be done by offsetting the long pointer carried by (B) so that, when the end of this pointer is opposite the pencil, the ordinate of (B)

will be K units greater than the ordinate of (P). The machine will then solve

$$\psi(x) = \int_a^x f_1(x)\,[\psi(x) + K]\,dx$$

That is, we solve the equation

$$\psi(x) - F(x) = \int_a^x f_1(x)\,\psi(x)\,dx$$

where $f_1(x)$ and $F(x)$ are known, and are related by

$$F(x) = K\int_a^x f_1(x)dx.$$

Another variation is as follows. Place the recording pencil on (B) instead of (P). Make the displacement between (B) and (P) always equal to (K) as before. If $\phi(x)$ is the curve traced by the pencil, it is evident that we solve

$$\phi(x) - K = \int_a^x f_1(x)\phi(x)dx.$$

Other variations may be introduced by making the displacement of (B) and (P) follow a known curve instead of simply remaining constant, and so on. By using also the multiplier, various new combinations become available.

As an example, suppose we set slider (A) at a constant value of unity, thus making

$$f_1(x) = 1$$

and suppose we place the pencil on (B) and keep (B) and (P) displaced by a distance (K). We then solve

$$\phi(x) - K = \int_a^x \phi(x)\,dx$$

One curve for $\phi(x)$ which will be obtained is evidently

$$\phi(x) = \varepsilon^x$$

which corresponds to

$$a = \log K.$$

These particular equations are of types readily solved by formal methods. By an extension of the scheme it has been possible to solve directly with the machine differential and integral equations of types which can be solved formally only with difficulty, or not at all. This subject is, however, too long to be treated in the present paper.

SUMMARY.

The mathematical instrument described above is a continuously recording integraph and multiplier. It will multiply together two curves, or will integrate the product, and will plot the resulting function. Integration and multiplication of the integral may be carried on simultaneously. It will also solve certain types of integral equations directly, and without the necessity of cut-and-try processes, or the evaluation of the terms of a series.

The Magnetic Susceptibility of Gases and Their Dependence upon Pressure and Temperature. ERWIN LEHRER. (*Ann. der Phys.*, **81**, No. 3, 1926.)—In the last two years A. Glaser has published results of his experimental work that seem to show a very strange magnetic deportment on the part of these gases, carbon dioxide, carbon monoxide, nitrogen and hydrogen. He measured the specific susceptibility of these gases as their pressure was reduced. This quantity is the ratio of the magnetic moment per unit of mass to the strength of the magnetic field producing the moment. He found that as the pressure grew less a change in the susceptibility manifested itself at a certain limiting pressure. For pressures below this the susceptibility became larger and for very small pressures approached a value thrice as much as at normal pressures. The purpose of the present paper is to examine the reliability of Glaser's results.

For the measurement of the susceptibility of the gases examined Quincke's method, as modified by Bauer and Piccard, was used. The gas was contained in a horizontal tube placed between the pole pieces of an electromagnet, the lines of force running perpendicular to the tube. The two ends of the tube were brought around to a manometer. When the field of the electromagnet was thrown on, there was no effect upon the manometer so long as the gas in the field was in the same state throughout. If, however, part of the gas was at a different temperature from the rest, the instrument showed a difference of pressure. Formulas are derived for testing Glaser's results and lengthy series of experiments were carried out. For the paramagnetic gas, oxygen, throughout the range 0° to 250° C. the law of Pierre Curie was verified that the product of the absolute temperature by the specific susceptibility is a constant. For the same range of temperature the susceptibility of argon and carbon dioxide was proved to be constant within 2 per cent. For hydrogen the divergence was as much as 5 per cent. No anomaly such as Glaser claimed to find was in any way indicated by the results.

Absolute values of the susceptibilities derived from the experiments are in satisfactory agreement with those obtained by Wills and Hector and by Také Soné. G. F. S.

ON THE PROPAGATION OF SOUND IN THE GENERAL BESSEL HORN OF INFINITE LENGTH.*

BY

STUART BALLANTINE.

Member of the Institute.

A FEW years ago the late Prof. A. G. Webster treated by an approximate mathematical method the theory of the propagation of sound in conical and exponential conduits of infinite length.[1] Webster's method, which is not by any means rigorous except for the conical horn, and then only when the radial vibrations are neglected, led to rather startling conclusions in the case of the so-called *exponential horn.* In the exponential horn the law of flaring, or growth, may be expressed as follows: *Area (at a point x on the axis)* $= A_0 e^{mx}$; x being the distance along the axis measured from the small end or throat. For this horn, using Webster's method, the same equations of propagation are obtained as for the exponential electrical line which has been diligently studied by Heaviside and others. Applying a sinusoidal pressure of single frequency to the throat of the conduit, it appears that as the frequency is varied from a high to a low value, the apparent velocity of the sound wave expressed by

$$vel. = \frac{V}{\sqrt{1 - \frac{m^2 V^2}{4\omega^2}}}, \qquad (V = vel.\ sound\ in\ air)$$

continuously increases and becomes infinite at the critical angular frequency: $\omega = mV/2$. At this frequency there is a cut-off in the transmission and the phase relation between pressure and velocity becomes such that no useful radiation of sound takes place below this frequency. Above the critical frequency, however, the pressure and velocity fall into phase coincidence at a much greater rate than in the case of the conical horn so that the transmission through the exponential horn is noticeably better as concerns that part of the low note spectrum which lies imme-

* Communicated by the Author.

[1] A. G. WEBSTER: *Jour. Nat. Acad. Sciences,* **5**, 275 (1919). I have unfortunately not had access to Webster's original publications, but have relied upon the reports of his work which have been given by A. N. Goldsmith and J. P. Minton, *Proc. Inst. Radio Engs.,* **12**, 423 (1924), who furnish the above reference.

diately above the cut-off frequency. This remarkable performance of the exponentially flared horn has led in recent years to its widespread adoption for electrophonic purposes, and indeed many persons have begun to evidence a reverential or sacrosanct attitude toward the magic law of flaring represented by $A = A_0 e^{mx}$. However, it seems unquestionable that the existence of a critical frequency is spurious and due only to the nature of Webster's approximate method and its failure near this frequency; spontaneously diffusive propagation at any frequency violates our physical intuition.

Feeling that there was no special esoteric significance attaching to the particular law of flaring in the exponential horn, but that the flaring was the thing, I began several years ago an investigation of the action of horns having other and more general types of flaring. In order to bring the investigation within the scope of a single analysis, and to secure as general and fundamental a law of flaring as possible, I chose the family of conduits represented by $Area = A_0 x^m$. In this group the flaring is adjustable by means of the index m, assumed for convenience to be integral. By Webster's method this law leads to equations for the sound propagation which are of the general Bessel's type; accordingly, it seemed appropriate to refer to the family of horns as *Bessel horns*. These are in analogy with Heaviside's *Bessel electric lines*, to the theory of which he devoted a considerable part of Vol. II, of his "Electro-magnetic Theory."

The derivation of the fundamental sound-wave equations has been systematically given by the late Lord Rayleigh in his treatise on the "Theory of Sound," Vol. II. This work has long been out of print and is, unfortunately, very scarce; moreover its nomenclature is at the present time somewhat archaic in view of the more abbreviated symbolism of the vector calculus. Ordinarily in bringing the equations to the canonical D'Alembertian form certain non-linear terms have been ignored which may require discussion in certain cases of wave propagation. In addition for many readers this will be a new subject, brought into present prominence by the recent rapid development of radio broadcasting and the use of horns for purposes of electrophonic reproduction. For these reasons it seems best to start at the beginning and derive the wave equation, avoiding unnecessary prolixity.

1. **Derivation of the Acoustic Wave Equation.**—Consider first the motion of an inviscid fluid under the action of a vector force $\boldsymbol{F}$, which is expressed by the equation:

$$\frac{d\boldsymbol{v}}{dt} + \frac{1}{\rho}\,\mathrm{Grad}\,P = \boldsymbol{F}. \tag{1}$$

Here $\boldsymbol{v}$ is the velocity, ρ the density, P the pressure; $\boldsymbol{F}$ is the force per unit mass. The type of motion to be considered is irrotational, that is to say, *Curl* $\boldsymbol{v} = 0$. In addition we have the equation of continuity:

$$\frac{\partial \rho}{\partial t} + \mathrm{Div}\,\rho\boldsymbol{v} = 0. \tag{2}$$

These equations determine the velocity, because a vector is uniquely determined by its *divergence, curl* and the boundary conditions; as to the pressure, a further relation between P and ρ is furnished by considering the thermodynamic process of compression. Two possible simple assumptions lead to the results:

$$\begin{aligned} &P = a^2\rho\,; && (\textit{isothermal, Boyle's law}) \\ &P = a^2\rho^\gamma. && (\textit{adiabatic}) \end{aligned} \tag{3}$$

In the first the gas obeys Boyle's law, the process of compression and expansion being isothermal; in the second this is considered as taking place adiabatically. γ is the ratio of specific heat at constant pressure to that at constant volume. The second assumption is usually taken to be more accurate, although for air γ is of the order of 1.4 and the difference is not serious.

Equation (1) is Euler's equation of motion and has the long-hand form:

$$\frac{d\boldsymbol{v}}{dt} + \frac{1}{\rho}\,\mathrm{Grad}\,P = \frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{2}\,\mathrm{Grad}\,\boldsymbol{v}^2 - \boldsymbol{v} \times \mathrm{Curl}\,\boldsymbol{v} + \frac{1}{\rho}\,\mathrm{Grad}\,P = \boldsymbol{F}. \tag{4}$$

For irrotation *Curl* $\boldsymbol{v} = 0$ and

$$\left.\begin{aligned} &\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{2}\,\mathrm{Grad}\,\boldsymbol{v}^2 + \frac{1}{\rho}\,\mathrm{Grad}\,P = \boldsymbol{F} && (1') \\ &\text{adding the other equations:} \\ &\frac{\partial \rho}{\partial t} + \rho\,\mathrm{Div}\,\boldsymbol{v} + \boldsymbol{v}\,\mathrm{Grad}\,\rho = 0 && (2') \\ &P = a^2\rho^\gamma. && (3') \end{aligned}\right\}$$

These three equations characterize disturbances of *finite* amplitudes. The first two are non-linear; the first contains, for

example, a term in v^2. The complicated nature of the sound waves described by these equations has been studied by Rayleigh,[2] Stokes, Riemann,[3] Weber and others. The more familiar theory of sound waves is that for waves of *infinitesimal* amplitudes, obtained by omitting from the equations all terms of higher order than the first. The equations then lead to the normal D'Alembertian wave equation for the propagation. In many recent papers on the theory of horns the investigation blithely starts from the D'Alembertian wave equation, apparently forgetting the assumptions which underlie it. In order to bring these into view we have started here at the beginning; arriving at this point, and realizing the simplicity which results from the omission of the higher order terms, it seems necessary to convince ourselves that in the applications contemplated these approximations are justifiable. What seemed particularly doubtful to me was the present application to horns, in which, due to the constriction at the throat, enormous velocities and pressures could occur, especially in outdoor public-address systems where considerable power might be employed.

The assumptions in question may be examined critically by considering the simple case furnished by a spherical wave in a conical conduit. The pressure p, and velocity v, are given by:

$$p = i\omega\rho\varphi\,, \tag{5}$$

$$v = \frac{1}{r}(1 + ikr)\,\varphi\,, \tag{6}$$

$$\varphi = A\,\frac{e^{-ikr}}{r}\,e^{i\omega t}\,; \tag{7}$$

$k = \omega/\text{vel. sound},$

r being the distance from the apex. At any point:

$$v = \frac{1}{i\omega\rho r}(1 + ikr)\,p\,. \tag{8}$$

Two cases may be considered: (1) At low frequencies where $kr \ll 1$ and (2) high frequencies where $kr \gg 1$. In the first case:

$$v = \frac{1}{i\omega\rho r}\,. \tag{9}$$

[2] LORD RAYLEIGH: "Aerial Plane Waves of Finite Amplitudes," *Proc. Roy. Soc.*, Series A, **84**, 247 (1910).

[3] RIEMANN-WEBER: "Die Partiellen Differentialgleichungen," **2**, Chapter 23, Vieweg, Braunschweig (1919).

Referring to equation (1′) we wish to compare the terms: $\partial v/\partial t$ and $\frac{1}{2}$ *Grad* v^2, the ratio of which is:

$$\frac{\partial v/\partial t}{\frac{1}{2}\,\text{Grad}\, v^2} = \frac{i\omega^2 r^2 \rho}{p}. \tag{10}$$

We see that the effect due to the second degree term increases rapidly as the throat is narrowed, as the pressure increases, and as the frequency is lowered. Let us take a numerical case: Assume that at a distance of 5000 cm. the pressure is 10 dynes cm.^{2} This will represent a fairly loud sound. Assume further that the throat is at a distance of 5 cm. from the apex. The pressure here will then be 10^4 dynes/cm.^{2}. For air $\rho = 1 \times 10^{-3}$ gm./cm.^{3} approximately. Then the ratio (10) $= \omega \times 2.5 \times 10^{-6}$. For a frequency of 320 cycles ($\omega = 2 \times 10^3$) the ratio equals 10 : 1; for a frequency somewhere near 100 cycles the v^2 term would be about equal to the other. We see that the assumption of linearity in the case of such a powerful source would not be without error, at least for the low frequencies. Non-linear distortion would be present and the sound wave would be of complicated form. The higher order effects may be reduced by enlarging the throat and reducing the velocity and pressure at this point. Where the horn is used in connection with ordinary loud-speaking telephone circuits in an ordinary room, the higher order effects are completely negligible unless the throat of the horn is unusually narrow. In the above case the ratio of terms would be about 100 : 1 at 320 cycles and 10 : 1 at 100 cycles. With these limitations in mind we introduce here the following approximations:

Assumption: The terms $\frac{1}{2}$ *Grad* v^2 *and* v *Grad* ρ *will be neglected in equations* (1′) *and* (2′), *respectively, and throughout we will retain only the first order effects.*

We now seek a general field scalar from which v and p may be derived by simple operations. *Curl* $v = 0$ is a necessary and sufficient condition for the existence of the scalar velocity potential φ such that:

$$v = -\,\text{Grad}\,\varphi. \tag{11}$$

Substituting this in equation (1′) we have:

$$\text{Grad}\left(\frac{P}{\rho} - \frac{\partial \varphi}{\partial t}\right) - P\,\text{Grad}\left(\frac{1}{\rho}\right) = F \tag{12}$$

Concerning the term $P\ Grad\left(\frac{1}{\rho}\right)$ we see from the relation (3) that if $\boldsymbol{v}\ Grad\ \rho$ is negligible this term will certainly be negligible; hence in free space when the impressed force $\boldsymbol{F} = 0$,

$$\text{Grad}\left(\frac{P}{\rho} - \frac{\partial\varphi}{\partial t}\right) = 0, \tag{13}$$

and

$$P = \rho\frac{\partial\varphi}{\partial t} + \text{const.} \tag{14}$$

The constant is the normal pressure P_0 in the undisturbed field. Let p represent the excess, or differential, pressure, that is $p = P - P_0$. From (3′)

$$\frac{\partial\rho}{\partial z} = \frac{\partial\rho}{\partial P}\frac{\partial P}{\partial z} = \frac{\rho}{pP}\frac{\partial P}{\partial z}. \tag{15}$$

Substituted in (2′) this gives for the first order effects:

$$\frac{\partial P}{\partial t} + \gamma P_0\ \text{Div}\ \boldsymbol{v} = 0. \tag{16}$$

The variation of P before the $Div\ \boldsymbol{v}$ term is ignored and P_0 substituted for it. By virtue of (11) and (14) this becomes:

$$\rho\frac{\partial^2\varphi}{\partial t^2} - \gamma P_0\ \text{Div (Grad}\ \varphi) = 0, \tag{17}$$

which is recognized as the standard D'Alembertian wave equation for φ:

$$\nabla^2\varphi - \frac{\rho}{\gamma P_0}\frac{\partial^2\varphi}{\partial t^2} = \nabla^2\varphi - \frac{1}{V^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{18}$$

where $V = Velocity\ of\ sound = \sqrt{\frac{\gamma P_0}{\rho}}$.

The excess pressure p and velocity $\boldsymbol{v}$ are derived from φ as follows:[4]

$$\left.\begin{aligned} p &= \rho\frac{\partial\varphi}{\partial t}, \\ \boldsymbol{v} &= -\text{Grad}\ \varphi. \end{aligned}\right\} \tag{19}$$

2. Derivation of the Equation of Propagation in the General Bessel Horn by Webster's Method. Solution in the Case of the Infinite Horn.—In the general Bessel horn as defined here the

[4] It may be noted that the signs of the derivatives given in Rayleigh's "Theory of Sound," Vol. 2, p. 24, are the reverse of these. The convention $A = -\text{Grad}\ \psi$ has more recently been adopted.

area A normal to the axis increases as a power m of the distance x along the axis from the small end; or

$$A = \text{Area} = A_0 x^m \quad \text{(Law of flaring in Bessel horn).} \tag{20}$$

We consider only the irrotational axial vibrations and assume no internal gas friction or friction between the gas and the walls of the horn. The walls are also considered to be rigid. According to Webster's method the suitable modification of the D'Alembertian equation (18) for the axial motion is:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial}{\partial x}(\log A)\frac{\partial\varphi}{\partial x} = \frac{1}{V^2}\frac{\partial^2\varphi}{\partial t^2}. \tag{21}$$

It may be remarked that the conical horn, which is a special case of the Bessel horn obtained by making $m = 2$, is the only member of the family to which this equation accurately applies. The felicity in this case is due to the fact that (21) then becomes the equation for a spherical wave and into such a wave a conical surface with apex at the source may be inserted without disturbing the symmetry of motion.

In the case of the Bessel horn (20), (21) becomes:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{m}{x}\frac{\partial\varphi}{\partial x} - \frac{1}{V^2}\frac{\partial^2\varphi}{\partial t^2} = 0. \tag{22}$$

Let $\varphi = ux^{-n}$, where $n = \frac{m-1}{2}$, then u satisfies the equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x} - \frac{n^2}{x^2}u - \frac{1}{V^2}\frac{\partial^2 u}{\partial t^2} = 0. \tag{23}$$

It is convenient at this point to adopt a symbolic notation: Let $\partial/\partial t = p$. In the case of applied sinusoidal pressures, $p = i\omega$ and the steady effects will vary as $e^{i\omega t}$. We have then $\partial^2/\partial t^2 = p^2$ and:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x} - \left(\frac{p^2}{V^2} + \frac{n^2}{x^2}\right)u = 0. \tag{24}$$

Letting $p^2/V^2 = k^2$ this equation has the solutions:

$$u = AZ_n(kx) + BZ'_n(kx), \tag{25}$$

where Z_n and Z'_n are linearly independent Bessel's functions. The complete solution is then:

$$\varphi = x^{-n}[AZ_n(kx) + BZ'_n(kx)]. \tag{26}$$

3. Theory of the Steady State. Effects Varying as $e^{i\omega t}$.— Thus far our equations are symbolic and the argument kx is of merely formal significance. The simplest and most important case is that in which the source is steady and sinusoidal and all

quantities vary with the time as $e^{i\omega t}$. The transient prelude to this state is reserved for treatment in a separate paper. In the steady state $p = \partial/\partial t = i\omega$ and $k = i\omega/V$; the argument is thus entirely imaginary and for convenience we will redefine k as representing ω/V.

In considering the theory of an *infinite* horn we are naturally anxious to obtain an expression in *wave solutions* rather than in terms of *normal solutions*. Out of the maze of Bessel's functions we find two, the Hankelian functions $H_n^{(1)}$ and $H_n^{(2)}$, which are particularly appropriate because of their direct physical significance as wave solutions. The first function, $H_n^{(1)}$, represents a wave moving from infinity toward the apex, or an *incoming cylindrical wave*, the second, $H_n^{(2)}$, represents a wave moving *outward* from the source. These functions are in analogy with the exponential functions e^{-x} and e^{x}; $H_n^{(2)}$ decreases monotonically and asymptotically to zero, and $H_n^{(1)}$ increases likewise to infinity, with x. For our theory the second function $H_n^{(2)}$, representing an outgoing wave, is of chief interest. In the case of the *finite* horn there is reflection from the mouth and both functions may be used in building up the wave solution. It may be noted that these functions are closely allied to the H_m and K_m functions whose fundamental importance as wave operators was recognized and emphasized by Heaviside.

The Hankel functions are expressed concisely in terms of the more familiar J_n and Y_n functions as follows:

$$\left.\begin{aligned} H_n^{(1)} &= J_n(x) + iY_n(x) \\ H_n^{(2)} &= J_n(x) - iY_n(x) \end{aligned}\right\}. \tag{27}$$

The appropriate solution for a wave moving outward from the source at the small end of the horn may then be completely expressed by:

$$\varphi = Ax^{-n}H_n^{(2)}(kx) = Ax^{-n}[J_n(kx) - iY_n(kx)]e^{i\omega t} \tag{28}$$

Adhering to integral values of m (flaring index, equation (20)), $n\left(=\frac{m-1}{2}\right)$ will advance by fractional steps of 1/2. The *order* of the Bessel's function corresponds to n and we shall henceforth refer to the corresponding horn as of this order. This should not be confused with the flaring index m. We can then make out the following schedule of solutions for the horns of various orders:

Schedule of Solutions.

Wave-functions for Velocity-potential in Outgoing Wave, Bessel Horns of Infinite Length: Area $= A_0 x^m$; Order n of Horn $= \frac{m-1}{2}$.

Order. $n = \frac{m-1}{2}$	Flaring Index m.	Formal Solution.	Expanded Form, $\varphi =$	Special Simplified Form where $n = k/2$.
0	1	$H_0^{(2)}(kx)\,e^{i\omega t} = [J_0(kx) - iY_0(kx)]\,e^{i\omega t}$		
$\frac{1}{2}$	2	$x^{-\frac{1}{2}} H_{\frac{1}{2}}^{(2)} e^{i\omega t} = x^{-\frac{1}{2}}[J_{\frac{1}{2}} + iJ_{-\frac{1}{2}}]\,e^{i\omega t}$		$= \kappa \frac{e^{-ikx}}{x} \cdot e^{i\omega t}$
1	3	$x^{-1} H_1^{(2)} e^{i\omega t} = x^{-1}[J_1 - iY_1]\,e^{i\omega t}$		
$\frac{3}{2}$	4	$x^{-\frac{3}{2}} H_{\frac{3}{2}}^{(2)} e^{i\omega t} = x^{-\frac{3}{2}}[J_{\frac{3}{2}} + iJ_{-\frac{3}{2}}]\,e^{i\omega t}$		$= \kappa \frac{e^{-ikx}}{x^2}\left(1 - \frac{i}{kx}\right) e^{i\omega t}$
2	5	$x^{-2} H_2^{(2)} e^{i\omega t} = x^{-2}[J_2 - iY_2]\,e^{i\omega t}$		
$\frac{5}{2}$	6	$x^{-\frac{5}{2}} H_{\frac{5}{2}}^{(2)} e^{i\omega t} = x^{-\frac{5}{2}}[J_{\frac{5}{2}} + iJ_{-\frac{5}{2}}]\,e^{i\omega t}$		$= \kappa \frac{e^{-ikx}}{x^3}\left[\frac{3}{kx} - i\left(\frac{3}{k^2x^2} - 1\right)\right] e^{i\omega t}$
3	7	$x^{-3} H_3^{(2)} e^{i\omega t} = x^{-3}[J_3 - iY_3]\,e^{i\omega t}$		
...	...	.....	.....	.....
n	$2n+1$	$x^{-n} H_n^{(2)} e^{i\omega t} = x^{-n}[J_n - iY_n]\,e^{i\omega t}$		

In making out this schedule advantage is taken of the relation:

$$H_n^{(2)}(x) = \frac{-i}{\sin n\pi}\left[e^{i\pi n} J_n(x) - J_{-n}(x)\right], \qquad (29)$$

when n is fractional. In cases where m is odd $2n$ is integral and these expressions for $H_n^{(2)}$ assume the particularly simple forms given in the last column.[5] The Y_n functions of this schedule are those adopted by Watson[6] as canonical functions of the second kind, after Weber, tables of which are given in his book.

4. General Case. Bessel Horn of Integral Order n; $A = A_0 x^{2n+1}$.—The Bessel family of horns divides into two parts: Those for which m is even, and those for which m is odd. In the latter case the order n is integral; in the former case it is half an integer and leads to simplified forms for the velocity

[5] JAHNKE and EMDE: "Funktionentafeln," p. 91, Teubner, Leipzig.

[6] G. N. WATSON: "Theory of Bessel Functions," p. 64, Cambridge, 1922.

potential, as noted above. In order to avoid the tedious and unnecessary printing of formulæ for the various cases under the second group, we will consider the general case. Later it will be useful to write down formulæ for the first few special cases.

The appropriate expression for the velocity potential is:

$$\varphi = Ax^{-n}[J_n(kx) - iY_n(kx)]e^{i\omega t}, \tag{30}$$

from which by (19):

$$p = i\omega\rho Ax^{-n}[J_n - iY_n]e^{i\omega t}; \tag{31}$$

$$v = \frac{A}{x^{n+1}}\left[\left(nJ_n - kxJ_n'\right) - i\left(nY_n - kxY_n'\right)\right]e^{i\omega t}. \tag{32}$$

We see that the pressure and velocity are in general not in phase, the discrepancy increasing as the frequency is decreased and the apex of the horn is approached. The flow of power is proportional to pv. This has two components, representing a real and an imaginary flow; the first represents actual conduction of sound energy from the source, and the latter the storage of energy in the reversible process of compression and in momentum. To discover the manner in which work is done by the source consider the pressure reaction on an imaginary piston situated in the throat at a point $x = x_0$. This piston is moving sinusoidally with the velocity $v_0 = V_0 e^{i\omega t}$. Then:

$$p = i\rho Vkx\,\frac{J_n - iY_n}{\left(nJ_n - kxJ_n'\right) - i\left(nY_n - kxY_n'\right)}\,V_0 e^{i\omega t}. \tag{33}$$

The piston works against two components of pressure, one of which, being in phase with the velocity, corresponds to useful work in the form of sound radiation, and the other, being in quadrature, represents an inertial or elastic storage of energy in compression. These pressure reactions as thus resolved may be considered as due to a resistance (R) and reactance (X), respectively, and the *acoustic throat impedance* of the horn is defined as $Z_0 = R_0 + iX_0$, or $p_0 = Z_0 v_0$. At any other point the impedance is similarly defined in terms of pressure and velocity. This impedance is the impedance per unit area; to find the total impedance we multiply by the area of the horn at that point. To find the resistance and reactance components from (33), we make use of the conjugate relation:

$$J_n(x)\,Y_n'(x) - Y_n(x)\,J_n'(x) = \frac{2}{\pi x}, \tag{34}$$

and the recurrence relation:

$$J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x), \tag{35}$$

also valid for Y_n. We have then:

$$\text{Resistance at throat} = R_0 = \rho V \frac{\frac{2}{\pi k x_0}}{\left(\frac{2n}{kx_0} J_n - J_{n-1}\right)^2 + \left(\frac{2n}{kx_0} Y_n - Y_{n-1}\right)^2}; \tag{36}$$

$$\text{Reactance at throat per unit area.} = iX_0 = i\rho V \frac{\frac{2n}{kx_0}\left(J_n^2 + Y_n^2\right) - \left(J_n J_{n-1} + Y_n Y_{n-1}\right)}{\text{(Same denominator)}}, \tag{37}$$

(Argument of Bessel functions $= kx_0$.)

Having these general formulæ we are now in a position to write down the expressions for the throat resistances and reactances of horns of other orders in this group.

5. Bessel Horn of Zeroth Order; Area $= A_0 x$.—This is the simplest case and the flaring may be said to be inverse, the shape of a horn of circular section being like that shown in Fig. 4. Putting $n = 0$ in (36) and (37), we get the components of throat impedance as follows:

$$R_0 = \rho V \frac{\frac{2}{\pi k x_0}}{J_1^2(kx_0) + Y_1^2(kx_0)}, \tag{38}$$

$$iX_0 = i\rho V \frac{J_0(kx_0) J_1(kx_0) + Y_0(kx_0) Y_1(kx_0)}{\text{(Same denominator)}}. \tag{39}$$

Since $Y_1(x \longrightarrow 0) = 2/\pi x$, and $J_1 \ (x \longrightarrow 0) = 0$, the resistance of this horn approaches zero with the frequency; as the frequency approaches infinity the resistance approaches asymptotically the value ρV per unit area.

The theory of this Bessel's horn of zeroth order has been considered by Goldsmith and Minton,[7] who refer to it as a *parabolic* horn. These authors arrive at the somewhat startling conclusion that the pressure and velocity are always 90° out of phase and hence there is no net transmission of sound through it. This spurious result is due to a faulty choice of Bessel's functions, whereby a standing wave system is produced instead of a progressive Hankelian wave. In such a normal solution there is of course no net flow of power unless one of the linearly independent solutions is provided with an imaginary coefficient.

[7] A. N. GOLDSMITH and J. P. MINTON: *Loc. cit.*, p. 475.

Physical intuition should guard against such an error, or the experimental fact that human beings have been blowing through such horns for 7000 years.

6. Bessel Horn of Order ½; Area = A_0x^3.—This is the conical horn and is a special member of the Bessel's family, $n = \frac{1}{2}$. The theory of this horn has been considered in detail by a number of investigators so that we shall merely note down the expressions for the throat resistance and reactance components to be used later in comparative computations. For the cone the velocity potential, pressure and velocity take the forms:

$$\varphi = A\frac{e^{-ikx}}{x}e^{j\omega t}, \tag{40}$$

$$p = i\omega\rho A\frac{e^{-ikx}}{x}e^{j\omega t} \tag{41}$$

$$v = A[1 + ikx]\frac{e^{-ikx}}{x^2}e^{j\omega t}, \tag{42}$$

giving for the components of impedance at the throat:

$$R_0 = \rho V\frac{k^2x_0^2}{1 + k^2x_0^2}, \tag{43}$$

$$iX_0 = i\rho V\frac{kx_0}{1 + k^2x_0^2}, \tag{44}$$

This is on the basis of the usual theory of the axial vibrations which has been given by Webster, Stewart and others; more complicated modes of vibration are possible, involving waves of non-axial type and these have been investigated theoretically by V. A. Hoersch,[8] a pupil of Professor Stewart.

7. Bessel Horn of Order Unity; Area = A_0x^3.—The horn of next higher order falls within the scope of the general equations (36) and (37). Introducing $n = 1$ into these equations yields the following equations for this case:

$$R_0 = \rho V\frac{\frac{2}{\pi kx_0}}{\left(\frac{2}{kx_0}J_1 - J_0\right)^2 + \left(\frac{2}{kx_0}Y_1 - Y_0\right)^2}, \tag{45}$$

$$iX_0 = i\rho V\frac{\frac{2}{kx_0}(J_1^2 + Y_1^2) - (J_0J_1 + Y_0Y_1)}{\text{(Same denominator)}}. \tag{46}$$

8. Bessel Horn of Order 3/2; Area = A_0x^4.—The simplified

[8] V. A. HOERSCH: *Phys. Rev.*, 2nd Ser., **25**, 218 (1925).

form for the velocity potential in this case is found in the schedule:

$$\varphi = A\left(1 - \frac{i}{kx}\right)\frac{e^{-ikx}}{x^2}e^{i\omega t}, \tag{47}$$

from which p and v are easily derived:

$$p = iA\rho\omega\left(1 + \frac{1}{ikx}\right)\frac{e^{-ikx}}{x^2}e^{i\omega t}, \tag{48}$$

$$v = A\left\{\frac{3}{x} - ik\left(1 - \frac{3}{k^2x^2}\right)\right\}\frac{e^{-ikx}}{x^2}e^{i\omega t}. \tag{49}$$

Finally for the throat resistance and reactance:

$$R_0 = \rho V \frac{\left(\frac{kx_0}{3}\right)^2}{1 + \left(\frac{kx_0}{3} - \frac{1}{kx_0}\right)^2}, \tag{50}$$

$$iX_0 = i\rho V \frac{\frac{2}{3}kx_0 + \frac{1}{kx_0}}{\text{(Same denominator)}}. \tag{51}$$

9. Bessel Horn of Order 2; Area = A_0x^5.—This is again a special case under the general formulæ so that we write down at once:

$$R_0 = \rho V \frac{\frac{2}{\pi kx_0}}{\left(\frac{4}{kx_0}J_2 - J_1\right)^2 + \left(\frac{4}{kx_0}Y_2 - Y_1\right)^2}, \tag{52}$$

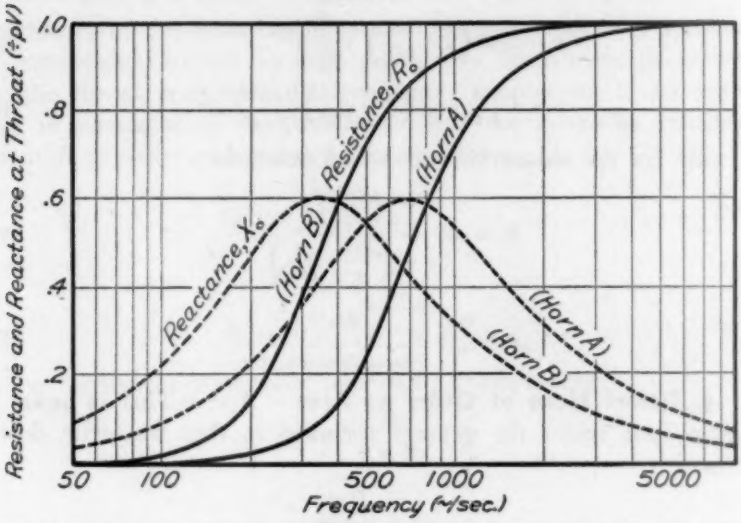
$$iX_0 = i\rho V \frac{\frac{4}{kx_0}\left(Y_2^2 + J_2^2\right) - (J_2J_1 + Y_2Y_1)}{\text{(Same denominator)}}. \tag{53}$$

In view of the increasing inaccuracy of the assumptions underlying the application of Webster's method, as the order n is increased and in order not to harass the attention with unnecessary minuteness, this compilation of formulæ will be terminated here. These few cases will suffice to illustrate the application of the general formulæ.

10. Computations and Curves. Bessel Horn of Order 3/2.—With this preliminary mathematical exercise, yielding the equations for the throat impedances for the first few values of m, we proceed with computations in a few practical cases. For specificity let us consider the Bessel horn of order 3/2, in which the flaring is given by $A = A_0x^4$. The resistance and reactance at the throat are shown graphically in Fig. 1, as a function of the

frequency of the applied pressure. Two different geometrical cases are taken: (A) In which the constant A_0 of the flaring equation is so chosen that starting with an initial opening of 1 cm. the horn diameter becomes 40 cm. at a distance of 100 cm.

FIG. 1.

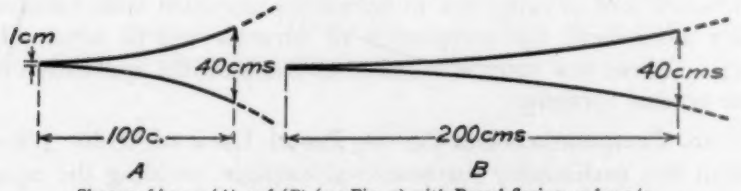


Resistance and reactance per unit area at throat of Bessel horn of order 3/2 (Area = A_0x^4) of infinite length. For shapes of horns designated (A) and (B) see Fig. 2.

from the throat $(x = x_0)$; and (B) in which the horn is 40 cm. in diameter at a distance of 200 cm. from the throat. The shape of these horns is shown in Fig. 2.

The greater acuteness of the (B) horn displaces the resistance

FIG. 2.

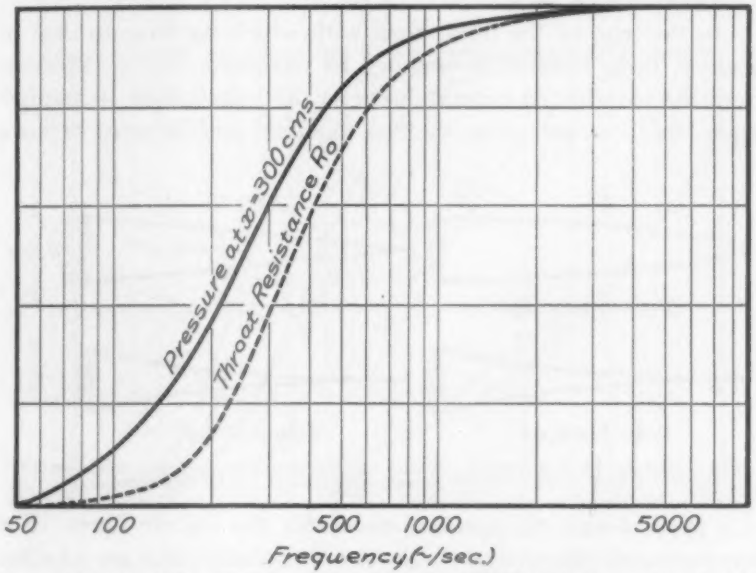


Shapes of horns (A) and (B) (see Fig. 1) with Bessel flaring, order 3/2.

curve toward the lower frequencies, for a given initial opening the distance between the apex $(x = 0)$ and the value of x corresponding to this opening is greater and thus k is smaller for the same resistance (or reactance).

Fig. 3 shows the pressure at a point of the axis in the (B) horn 300 cm. from the throat, as a function of frequency. The velocity at the throat is assumed to be constant. So far as its variation with frequency is concerned, the pressure at a distance

FIG. 3.



Curves of throat resistance R_0 and sound pressure at a point of the axis 300 cm. from throat in Bessel horn of 3/2 order (horn B, Fig. 2).

from the throat is proportional to the pressure at the throat. This follows from the fact that for this horn:

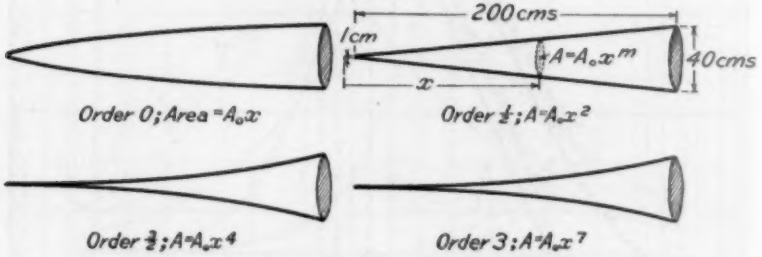
$$p = p_0 \left(\frac{1 + \frac{1}{ikx}}{1 + \frac{1}{ikx_0}} \right) \frac{x_0^2}{x^2} e^{-ik(x - x_0)} e^{i\omega t}$$

and when x is large $kx \gg 1$, so that the modulus of this expression reduces to a constant for all important frequencies. Now the pressure at the throat is equal to the product of the acoustic throat impedance and the velocity. The velocity being constant the curve in Fig. 3 is very approximately a duplicate of the curve of throat impedance ($\sqrt{R_0^2 + X_0^2}$), although it follows the general trend of curve of throat resistance. It is generally true of infinite horns that at a distance from the throat the pressure is

proportional to the pressure at the throat. The "frequency characteristic" of such a horn will thus depend upon the mode of its excitation; if the throat pressure is maintained constant, the distant pressure will be uniform over the frequency range. The more technical subject of excitation will be considered in a contemporaneous paper.

In the case of the finite horn with which we have to deal in practice, these results are modified by resonance due to reflection from the mouth. In general, however, as I shall show in another paper, the pressure curve as thus modified and serrated follows

FIG. 4.



Family of Bessel horns of various orders for which computations of throat resistance are shown in Fig. 5.

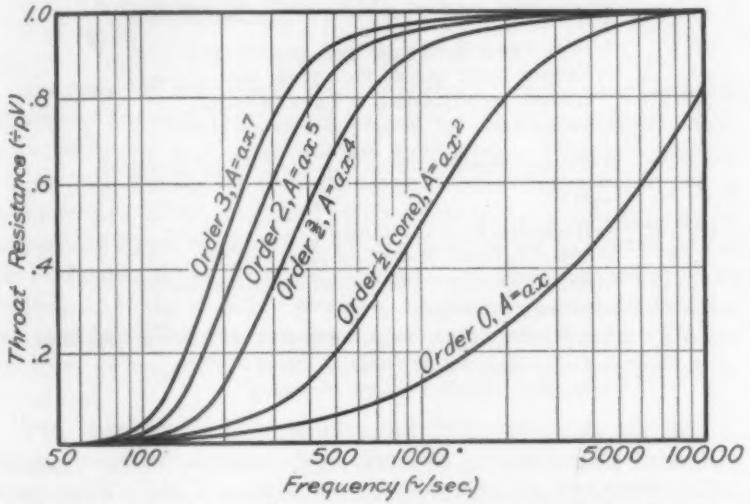
in a general way the pressure curve for the infinite horn. For this reason the theoretical results for the infinite horn are of some practical value as generally indicating the performance of the corresponding finite horn.

11. Comparison of Horns of Different Orders. Effect of Flaring.—We now come to the climax and important part of the investigation, the study of the effect of flaring. This is investigated by comparing the resistance curves for horns of successively increasing flaring indices, starting with $m = 1$. As a basis of comparison, we imagine the infinite horns so selected that if they were finite their axial lengths, initial and final openings would be identical. In other words, the surfaces representing the initial and final openings are to be connected by surfaces of the type $A_0 x^m$, the index m being subject to variation. The family of Bessel horns which has been computed is illustrated in Fig. 4.

The throat resistance for these horns is shown graphically in Fig. 5, plotted as usual against frequency. The flaring indices are 1, 2, 4, 5 and 7, $m = 2$ representing the cone.

The effect of increasing Bessel flaring is to augment the response for low notes in quite the same fashion as in the exponential horn, except that no critical frequency exists. The flaring of the horn is thus of advantage in promoting a more uniform frequency response and a better quality of conduction from throat to mouth. In view of these results the exponential horn loses a

FIG. 5.



Summary of results. Throat resistance for infinite Bessel horns of ascending orders, showing effect of increasing flaring. For shapes of horns see Fig. 4.

great deal of its esoteric significance. As expected, the flaring seems to be the thing.

It is a common characteristic of all horns excited by a constant velocity source that the propagation of the low-frequency vibrations is discouraged as the source approaches the apex of the horn. This is due to the pressure and velocity falling into the quadrature relationship noted above. In order to retain the transmission of the lower notes, x_0 may be increased; this will increase the area of the throat, or for a given throat area will require a horn of greater acuteness.

I am indebted to Profs. G. W. Pierce and J. C. Slater, of Harvard, for friendly comment upon the first manuscript of this paper.

WHITE HAVEN, PENNA.,
October 12, 1925.

LIST OF SYMBOLS.

φ	= velocity potential.
p	= excess or differential pressure.
P_0	= normal pressure.
P	= total pressure $= P_0 + p$.
γ	= specific heat at constant P/specific heat at constant V.
ρ	= density of gas.
v	= velocity.
V	= velocity of sound $= \sqrt{\frac{\gamma P_0}{\rho}}$.
A	= cross-sectional area of horn.
x	= distance along axis of horn from apex.
x_0	= distance corresponding to throat opening.
m	= flaring index for Bessel horn, Area $= A_0 x^m$.
n	= order of Bessel horn $= \frac{m-1}{2}$, or Bessel function.
k	$= \omega/V$.
$H_n^{(1)}, H_n^{(2)}$	= Hankelian Bessel functions.
$J_n(x)$	= Bessel function of the first kind.
$Y_n(x)$	= Bessel function of the second kind after Weber.
$Z = R + iX$	= acoustic impedance $= p/v$.
R, X	= resistance and reactance components of acoustic impedance.
Z_0, R_0, X_0	$= Z$, R and X per unit area at throat of horn.
ω	= angular frequency $= 2\pi \times$ *frequency*.

Rôle of Chemistry in Modern Agriculture.—W. M. JARDINE, U. S. Secretary of Agriculture (*J. Asso. Official Agric. Chem.*, 1926, **9**, 460–461), recognizes chemistry as one of the most fundamental of the sciences upon which agriculture is based. He stresses the application of chemistry in the more diversified utilization of crops as an important service in the immediate future. Examples of such utilization are the manufacture of citrus by-products from cull oranges and lemons, and the manufacture of both sugar and syrup from the sugar cane. The development of varied methods for the utilization of crops will benefit producer, distributor, and consumer.

J. S. H.

IN CONNECTION with the celebration of the fiftieth anniversary of Johns Hopkins University, a dinner was given by the Faculty of Physics to former students in that department. One of the interesting features at this occasion was an account by Prof. Edwin H. Hall, Ph.D., '80, now in Harvard University, of his visit to the very spot in the old laboratory of physics where he discovered the Hall Effect. Prof. R. W. Wood, after an account of his recent experiments in the production of waves of exceedingly high frequency in liquids, delighted his auditors with some of his witty scientific verses.

G. F. S.

PHOTOGRAPHIC MEASUREMENT OF RATE OF DETONATION OF EXPLOSIVES.*

BY

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DEFINITION AND METHODS USED.

THE rate of detonation is an important factor in the choice of explosives for different kinds of work. It is sometimes called the velocity or quickness of the explosives. Other conditions being equal, the higher the rate of detonation the greater will be the shattering effect produced by the explosive. In quarrying, for example, if large blocks of stone are desired, an explosive of low rate is chosen, but if the work requires finer stone for use in stone crushers or if the rock is extremely tough, an explosive of higher rate is used. The rate of detonation, along with other factors, furnishes an excellent guide for deciding upon the type of explosive to use.

Two methods are commonly used in determining the rate of detonation—one devised by Mettegang [1] and the other by Dautriche.[2] The Mettegang recorder, as the instrument is called, records on a rapidly revolving smoked drum, sparks produced by the breaking of fine wires carrying current, which are threaded through the explosive at known distances apart. The Dautriche method employs Cordeau-Bickford detonating fuse whose rate has been determined previously by means of the Mettegang recorder.

The condenser discharge method has recently been described by Wagstaff [3] and by Jones [4]; it also depends upon the breaking of fine wires carrying current. Smith [5] describes a method that employs spark photography of sound waves produced by detonation of the explosive.

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* Communicated by the Authors.

[1] METTEGANG, HANS, "Fifth Int. Congress of Applied Chemistry," **2**, 1903, p. 322.

[2] DAUTRICHE, H., *Compt. Rend.*, **143**, 1906, p. 641.

[3] WAGSTAFF, J. E. P., *Proc. Roy. Soc.* (London), **105A**, 1924, p. 282.

[4] JONES, E., Safety in Mines Research Board, Paper No. 22, 1926, London.

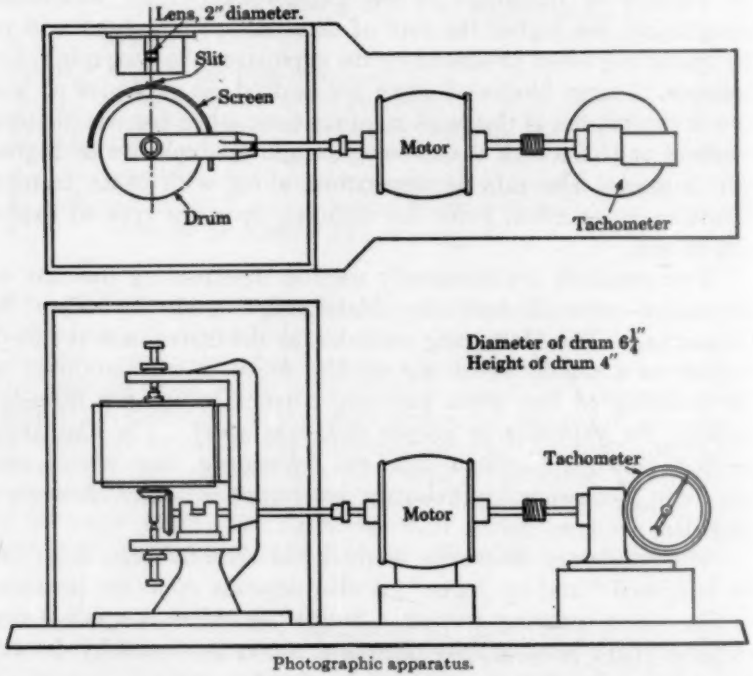
[5] SMITH, J. E., *Phys. Rev.*, **25**, 1925, pp. 870–876.

Factors such as the lag in breaking an electrical circuit by the detonation of an explosive, as found by Kast,[6] and the current leakage through the gaseous products of detonation measured by Wagstaff show that the determination of the rate of detonation by electrical method is not absolutely the true rate of detonation in all cases. At best it is the average rate of detonation between two points not considering any errors of measurement.

PHOTOGRAPHIC METHOD OF DETERMINING RATE OF DETONATION.

The photographic method described in this paper has the advantage that a permanent record is produced which shows the

FIG. 1.



Photographic apparatus.

rate of detonation at every point in the file of explosive. Dixon[7] has employed the method for determining the rate of detonation of explosive gas mixtures. Laffitte[8] has determined the rate of

[6] KAST, H., "Spreng. und Zunstoffe," p. 1025, Vieweg. (1909).

[7] DIXON, H. B., *Phil. Trans.*, Series A, **200**, 1903, p. 315.

[8] LAFFITTE, P., *Compt. Rend.*, **178**, 1924, pp. 1277–1779; *Ann. des Phys.*, **4**, 587–694 (1925).

detonation of explosives in small glass tubes, 5 to 15 mm. in diameter. Experiments were begun at the Explosives Experiment Station, of the Bureau of Mines, Bruceton, Penna., in October, 1925, in which photographs of columns of explosive, 7/8 inch to 1¼ inches in diameter by 6 feet long, were taken on a moving film. A study of the rate of propagation of detonation across the air-gap between two cartridges was also undertaken.

The photographic apparatus shown in Fig. 1 consists of a rotating drum around which was wrapped a sensitized film. A quartz lens focusing through a stenopaic slit to the drum was used to obtain an image of the flame of the detonation. In all the determinations the drum was rotated at a peripheral velocity of 20 metres per second, which is about the limit of the apparatus. Tests were conducted at night and the image of the flame on the explosive recorded on the film by means of its own light. A column of explosive consisting of dynamite cartridges with the crimped ends cut off, placed end to end and wrapped in two thicknesses of Manila paper, was suspended from an overhead vertical wire at a distance of 20 feet from the camera. The length of the column employed in the various determinations varied from 30 to 70 inches and the diameter from 7/8 inch to 1¼ inches. The charge was detonated from the bottom of the column by means of a No. 6 electric detonator.

METHOD OF CALCULATION.

As the film moves horizontally, and the explosive does not detonate at an infinite rate, the image of the flame will not be vertical but inclined at an angle, whose magnitude will depend upon the rate of detonation. The lower the rate of detonation, the greater will be the inclination of the image from the vertical.

In the present apparatus a vertical distance of 31 millimetres on the film represented a length of explosive of 1 metre, and with the film travelling at a peripheral speed of 20 metres per second it can be easily shown that the rate of detonation in metres per second equals 645 tan θ, where θ is the angle which the image of the flame makes with the horizontal. Determinations may be made either by measuring the angle θ or by constructing a right-angled triangle with the straight edge of the flame image as the hypotenuse, and calculating tan θ directly.

RATES OF DETONATION OF DYNAMITES.

Determinations by the above method were made on four ammonium nitrate-nitroglycerin explosives, 40 per cent. straight nitroglycerin dynamite and 40 per cent. ammonia dynamite.

FIG. 2.

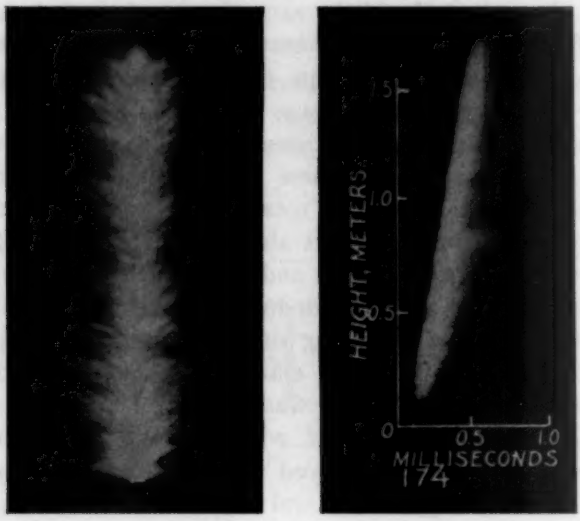


TABLE I.

Comparison of Rate of Detonation Determinations by Photographic Method and by Mettegang Recorder.

Type of Explosive.	Rate of Detonation, Metres per Second.			
	Photographic Method (Separate Results).	Average.	Mettegang Recorder (Average).	Dimensions of Explosive—Column, Inches.
*Permissible A.....	2440 2620	2530	2490	⅞ × 67
*Permissible B.....	2960 2980 3010	2980	2980	1¼ × 30
*Permissible C.....	2200 2240 2010	2150	2140	1¼ × 30
*Permissible D.....	2100 1980 2050	2040	2000	1¼ × 30
40 per cent. nitroglycerin dynamite...	3830 4270 4500	4170	4020	⅞ × 70
40 per cent. ammonia dynamite.....	3480 3490	3490	3570	1¼ × 30

* Explosive, permissible for use in coal mines.

FIG. 3.

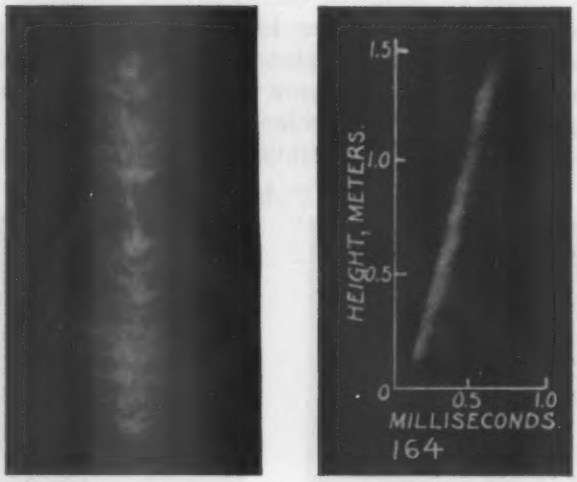
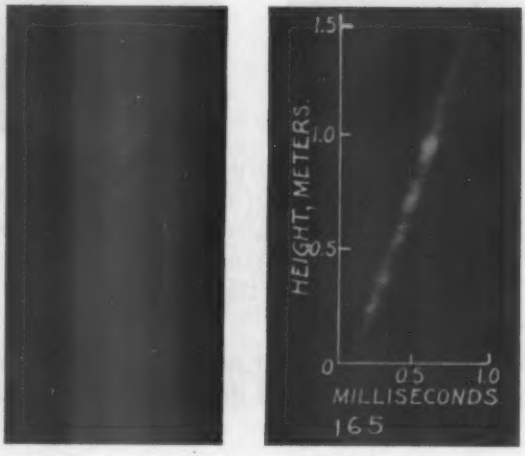
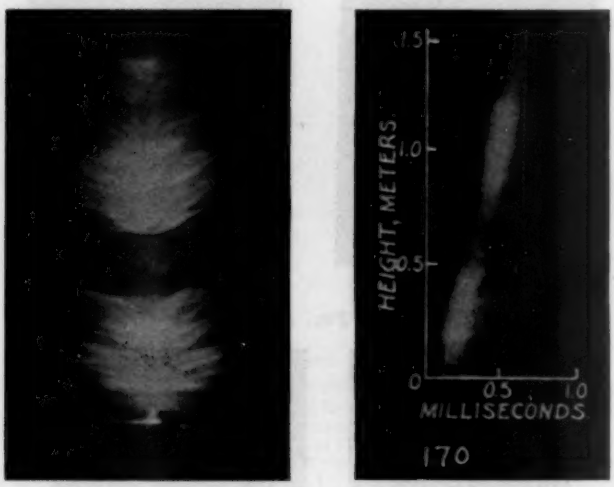

1.5
1.0
0.5
0
HEIGHT, METERS.
0.5
1.0
MILLISECONDS.
164

FIG. 4.


1.5
1.0
0.5
0
HEIGHT, METERS.
0.5
1.0
MILLISECONDS
165

Figs. 2, 3, and 4 show typical photographs taken both on a moving film and by means of an ordinary camera on a fixed plate. In Table I are set down the rates of detonation determined by the photographic method and for comparison results on similar charges obtained by means of the Mettegang recorder. Results are the average of two to three determinations. It will be seen that the determinations by the photographic method on explosives having rates of detonation varying from 2000 to 4200 metres per second agree very well with similar determinations made with the Mettegang recorder. An apparatus has since been built capable

FIG. 5.



of giving film speeds of 50 metres per second and correspondingly increased accuracy. In the case of determinations on 40 per cent. straight dynamite where the values varied considerably, these variations represent actual differences in the rate at which the column of explosive detonated in the different tests and not errors in measurement.

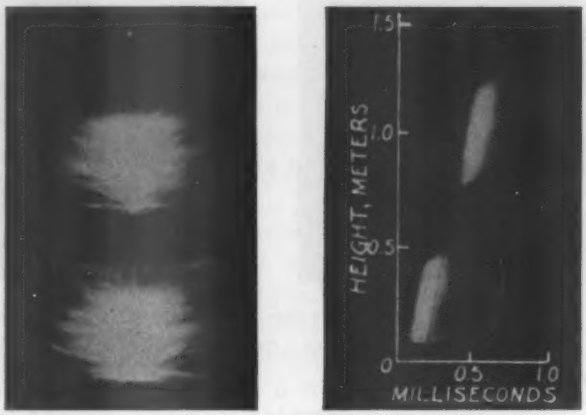
COLUMNS CONTAINING EXPLOSIVES OF DIFFERENT RATES OF DETONATION.

Fig. 5 shows a photograph of the detonation of a file of eight cartridges in which the first two cartridges were 40 per cent. straight nitroglycerin dynamite, the next two permissible A, the next two 40 per cent. straight nitroglycerin dynamite, and the last two permissible A. Detonation was from the bottom of the first cartridge of 40 per cent. straight dynamite. The position of

the two types of explosive is evident from the variation in light intensity on the photographs and by the sharper angle with the vertical made by the 40 per cent. dynamite section of the photograph. Calculation shows an average rate for 40 per cent. straight nitroglycerin dynamite of 4260 metres per second, and for permissible A 2440 metres per second. The high rate of detonation of the 40 per cent. straight nitroglycerin dynamite is evidently not imparted to the slower detonating permissible but each detonates at its own characteristic rate.

An interesting result of this method of combination is shown

FIG. 6.



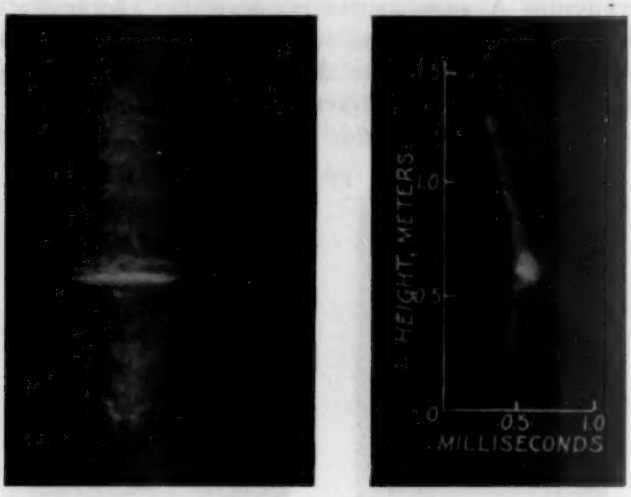
in Fig. 6, which is a photograph of the detonation of a file of eight cartridges consisting alternately of two sticks of 40 per cent. straight nitroglycerin dynamite and two sticks of permissible E, virtually a flameless explosive when detonated in the open. The photographic plate is not affected by the detonation of permissible E, with the result that only the flame from the 40 per cent. dynamite shows on the film but the effect of the intervening cartridges of permissible E is evidenced by the fact that the images of the two sets of 40 per cent. dynamite cartridges are not in a straight line.

DETONATION SIMULTANEOUSLY FROM BOTH ENDS.

Fig. 7 is a photograph of a column of permissible A, 7/8 inch diameter and 60 inches long, detonated from opposite ends simultaneously by means of two No. 6 detonators connected in parallel

and fired from a 220-volt d. c. line with a capacity of 125 amperes. It is interesting to note the bright line which is the result of the meeting of the detonation waves and the consequent higher temperature caused by compression of the gaseous prod-

FIG. 7.



ucts. The detonators did not explode at the same instant, hence the detonating waves did not meet at the exact centre of the column.

SUMMARY.

(1) A photographic method of determining rate of detonation of explosives has been described.

(2) Photographic determinations of the rate of detonation of explosive are essentially the same in value as determinations made with the Mettegang recorded.

(3) When the detonation of a permissible explosive of the ammonium nitrate-nitroglycerin type is initiated by an explosive detonating at a much higher rate, the characteristic rate of detonation of the permissible explosive is not changed.

(4) Some types of permissible explosives are flameless when detonated in the open.

THE CONTRAST OF PHOTOGRAPHIC PRINTING PAPER.*†

BY

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SECTION D. DERIVATION OF CONTRAST FROM SENSITOMETRIC AND STATISTICAL DATA.

IN THE preceding sections a detailed account of experimental work resulting in the determination of the characteristics of negatives which give the best possible photographic prints (optimal positives) on various positive materials has been given. The characteristics of the prints made from these negatives (optimal negatives) and the sensitometric curves and constants of the positive materials used have also been determined. We are now in a position to proceed with the correlation of the statistical and sensitometric results and to compute the contrast values for the positive materials.

The first step in this process is to determine for each of the positive materials used just what part of the available density scale of each material was employed in making the *optimal positives*. It is obvious that the characteristics of the optimal positives and the optimal negatives cannot afford any information relating to the positive materials as a whole, unless the entire available density or exposure ranges of these materials were utilized in making the optimal positives. It will be seen that this was not the case in general, hence any evaluation of contrast based upon numerical values derived from the optimal positives and negatives must of necessity apply only to the portions of the characteristic curves actually utilized. This leads at once to the conclusion that in speaking of contrast, it will be necessary to differentiate between *partial* and *total* contrast values. The contrast of the various materials as evaluated from the standpoint of the particular object (Willow Pond, Fig. 5) used in this work must be a function of that portion of the sensitometric curve which was actually used and may not be equal to, or even proportional to, the *total avail-*

* Communicated by Dr. C. E. K. Mees, Director of Laboratory, and published as Communication No. 264 from the Research Laboratory of the Eastman Kodak Company.

† Continued from p. 625 of the November issue.

able contrast or *contrast capacity* of the material. It is necessary, therefore, to isolate that portion of the characteristic curve actually used, and from its shape and extent to compute the values of contrast effective or available for the reproduction of the particular object, which was chosen as being typical of the normal landscape.

Following this a further analysis of the data will be given in an attempt to establish certain relationships between the characteristics of the *optimal negative, optimal positive, positive material* and *object* which determine the suitability of various positive materials for the reproduction of negatives of various types.

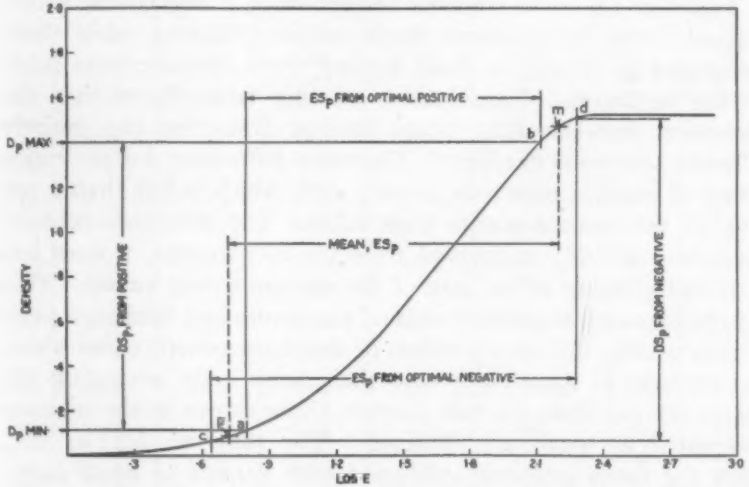
I. ISOLATION OF THE UTILIZED SECTION OF THE CHARACTERISTIC CURVE.

The method of procedure will be described by referring to Fig. 30, which illustrates the way in which the *used portion* of the characteristic curve is localized. The curve shown is a typical density-log E characteristic for a positive material and is the result of sensitometric work described in the previous section, such a curve having been obtained for each of the positive materials used. It will be recalled that the statistical treatment of the subject gave values of maximum and minimum density for the optimal positives. These values were derived by interpolation from the density measurements made on the actual prints and in Table XVIII these maximum and minimum density values for each optimal positive are given. Using the characteristic curve for a given material, the maximum and minimum densities of the optimal print made on that material are located on the density scale as indicated in Fig. 30 (points marked D_p max. and D_p min.). Horizontal lines from these points establish the points a and b on the characteristic curve. Perpendiculars dropped from the points a and b determine the limiting values of log exposure. The log E interval between these values gives the exposure scale of the print, ES_p, *as determined from measurements made on the actual prints.*

Data relative to the magnitude of the exposure scale, ES_p, of the optimal positive derived from an entirely independent source are also available. These are deduced from the values of the density scale, DS_n, of the optimal negatives and were determined by interpolation from density measurements made on the negatives actually used. From the requirements of tone repro-

duction theory (Fig. 28) the exposure scale of the positive must be equal to the density scale of the negative from which the positive was made. Referring again to Table XVIII it will be seen that in the fourth column (DS_n) values of density scale for the optimal negatives are given. When these values were compared with the exposure scale values as derived from the measurements made on the optimal prints, some discrepancies were found. Now there is no reason to believe that the data derived from the prints are any more reliable than those deduced from the negatives. It was considered advisable, therefore, to give each of

FIG. 30.



Showing graphically the method employed for arriving at the most probable values of exposure scale, ES_p, and density scale, DS_p, using data derived both from the optimal negatives and the optimal positives.

these values equal weight. The data derived from the measurements of negative density of course give no information as to the *position* of the optimal print on the sensitometric curve. It is necessary, therefore, to rely entirely on information derived from the positives for locating the *position* of the utilized exposure scale on the characteristic curve. In order to give equal weights in the determination of the *magnitude* of ES_p to the data derived from the measurements made on the negatives, the value derived from the negative was laid off as indicated in the figure, being symmetrically placed with respect to the log E interval between the points a and b. In this way the points c

and d on the characteristic curve were established. Now if the log E interval between the points b and d is bisected, the point m, as indicated, is established, and likewise the bisection of the log E interval between c and a establishes the point p. The log E interval between m and p now becomes the mean exposure scale of the optimal positive as determined by measurements made both on the optimal negatives and positives, and the density interval between the points m and p becomes the most probable value of density scale.

The procedure as illustrated in Fig. 30 was followed for each of the positive materials used. The results obtained are tabulated in Table XXIX. The columns designated by P contain the values derived from measurements made on the positives, while those designated as N contain those derived from measurements made on the negatives. Examination of this table shows that the agreement between ES_p values derived from the two entirely different sources is excellent. The mean difference, for the entire group of positive materials, is only .028, which is less than 2 per cent. of the mean exposure scale value. The difference between the values of DS_p, as derived from the two sources, is even less than that existing in the case of the exposure scale values. This is to be expected since the points of maximum and minimum print density usually fall upon portions of the characteristic curve where the gradient is appreciably less than unity. By averaging the values derived from the two sources, those shown in the columns designated as *mean* are obtained. The ratio of DS_p to ES_p gives the mean gradient, *evaluated with respect to equal increments of log E,* of that portion of the characteristic curve utilized in making the optimal positive. This ratio is called the *mean gradient* of the optimal positive and is designated by the symbol ϕ_p. Expressed formally the value of ϕ_p is given by the equation

$$\phi_p = \frac{1}{\log E_2 - \log E_1} \int_{\log E_1}^{\log E_2} \frac{dD}{d \log E} \, d \log E \qquad (1)$$

$$\phi_p = \frac{D_2 - D_1}{\log E_2 - \log E_1} = \frac{DS_p}{ES_p} \qquad (2)$$

D_2 = Maximum density of optimal positive.
D_1 = Minimum density of optimal positive.
Log E_2 = Maximum log E of optimal positive.
Log E_1 = Minimum log E of optimal positive.

TABLE XXIX.

Values of Density Scale and Exposure Scale for the Optimal Positives as Derived Independently from Measurements Made on the Optimal Positives and Optimal Negatives, Together with the Mean Values as Determined from These Sources.

No.	DS_p.			ES_p.			ϕ_p.
	P.	*N.*	Mean.	*P.*	*N.*	Mean.	Mean.
1	1.05	1.06	1.06	1.77	1.80	1.78	.59
2	1.06	1.08	1.07	1.51	1.55	1.53	.70
3	1.05	1.06	1.06	.83	.84	.84	1.26
4	1.31	1.31	1.31	1.64	1.62	1.63	.80
5	1.26	1.26	1.26	1.40	1.40	1.40	.90
6	1.26	1.28	1.27	.99	1.03	1.01	1.26
7	1.20	1.18	1.19	.82	.80	.81	1.48
8	1.43	1.43	1.43	1.70	1.69	1.70	.84
9	1.44	1.43	1.44	1.36	1.33	1.34	1.07
10	1.40	1.40	1.40	.96	.96	.96	1.46
11	1.38	1.36	1.37	.73	.69	.71	1.93
12	1.44	1.45	1.44	1.14	1.17	1.16	1.24
13	1.34	1.32	1.33	.72	.69	.70	1.90
14	1.13	1.13	1.13	1 33	1.30	1.32	.86
15	1.04	1.03	1.04	.95	.93	.94	1.11
16	1.35	1.35	1.35	1.29	1.29	1.29	1.05
17	1.24	1.23	1.24	.89	.88	.88	1.41
18	1.41	1.42	1.42	1.01	1.04	1.02	1.39
19	1.36	1.34	1.35	.73	.70	.72	1.88
20	1.08	1.10	1.09	1.28	1.35	1.32	.83
21	1.32	1.34	1.33	.58	.60	.59	2.26
22	1.37	1.41	1.39	1.35	1.42	1.38	1.01
23	1.11	1.13	1.12	1.49	1.53	1.51	.74
24	1.10	1.10	1.10	1.53	1.53	1.53	.72
25	1.10	1.13	1.12	1.38	1.43	1.40	.80
26	1.08	1.08	1.08	1.53	1.52	1.52	.71
27	1.32	1.29	1.30	1.25	1.19	1.22	1.06
28	1.32	1.31	1.32	1.15	1.13	1.14	1.16
29	1.32	1.28	1.30	.86	.80	.83	1.57
30	1.13	1.13	1.13	1.49	1.49	1.49	.76
31	1.21	1.23	1.22	1.42	1.44	1.43	.85
32	1.25	1.26	1.26	1.40	1.42	1.41	.89
33	1.10	1.09	1.10	1.72	1.70	1.71	.64
34	1.36	1.40	1.38	1.45	1.51	1.48	.93
35	1.18	1.17	1.18	1.74	1.69	1.72	.69
36	1.39	1.37	1.38	1.34	1.31	1.32	1.04
37	1.07	1.08	1.08	1.49	1.53	1.51	.72

The values of ϕ_p are given in the last column of Table XXIX.

In the later work on the computation of contrast, it will be necessary to evaluate the mean gradient (the average or mean value of $dD/d \log E$) *with respect to equal increments of density* and care should be taken not to confuse the value of ϕ_p as defined above with the later evaluation of mean gradient.

The actual values of maximum and minimum density for the optimal positives are of considerable interest and the most probable values of these factors were determined by reading the ordinates of the points m and p as illustrated in Fig. 30. These most probable values of D_p max. and D_p min. are given in Table XXXI. It is interesting to note that the value of minimum positive density (D_p min.) is not constant but increases systematically with the *contrast* of the material. For instance, in the first group (1, 2, and 3) the value of minimum density increases from .06 to .11, while in the second group (4 to 7) the minimum density increases from .07 to .16, and in the third group (8 to 11) the value goes up to .20. In each of these groups *contrast* increases progressively with the number, the first group being a paper having a matte surface, the second a semi-matte surface, and the third a glossy surface. A rule for the determination of correct exposure in printing is frequently quoted, which states: "The printing exposure should be such as to give a just perceptible density in the print for the portion of the negative having maximum density." It is obvious that this condition is not fulfilled in these optimal prints. The greater the contrast of the paper, the greater is the minimum density. There seems to be some reason, therefore, which prevents the utilization of as great a percentage of the available density scale in the case of the contrasty materials as can be utilized in the case of the less contrasty materials. We are lead from this to inquire just what controls this factor and in the following pages a careful analysis of this will be made.

II. DETERMINATION OF LIMITING GRADIENT, ϕ.

Having localized for each of the positive materials the part of the characteristic curve utilized in making the optimal positives, we shall now proceed to examine carefully the values relating thereto. It is at once evident that the maximum and minimum densities do not in general fall at points on the characteristic curve where the gradient $G_y = dD/d \log E = .2$; nor in fact do they fall.

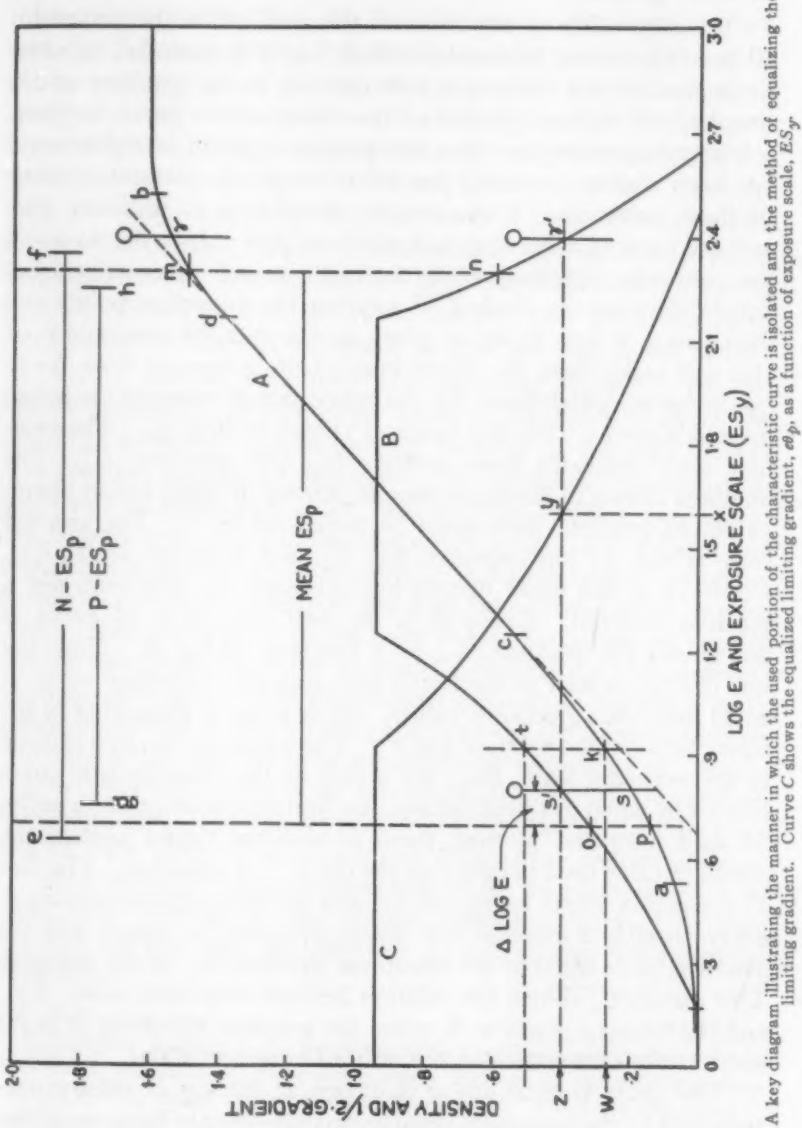


FIG. 31.

A key diagram illustrating the manner in which the used portion of the characteristic curve is isolated and the method of equalizing the limiting gradient. Curve *C* shows the equalized limiting gradient, ϕ_p, as a function of exposure scale, ES_y.

in the case of positive materials differing in contrast, at points of equal gradient.

It is impossible to reproduce in this publication the curves for all the thirty-seven materials studied, but it is essential, in order to emphasize this important fact relating to *the gradient at the limits of the utilized portion* of the characteristic curve, to show a few typical examples. For this purpose a group of eight curves has been chosen, covering the entire range of contrast existing in these materials. These curves are plotted to uniform scale and are marked according to a uniform plan. In order to avoid the confusion resulting from the inclusion of all constructional details showing the method of locating the important points and dimensions, a key figure is given and a detailed description of this will make clear the significance of the curves in Figs. 32 to 39, inclusive, which show the characteristics of some of the actual materials used. This key figure is shown in Fig. 31. The ordinate scale indicates both density, D_y, and gradient, G_y. The numbers shown in the figure denote density directly, but to obtain values of gradient these must be multiplied by 2. The abscissa scale is that of log exposure, log E_y, and exposure scale, ES_y. Curve A is the usual density-log exposure characteristic for a positive material. Curve B is the first derivative of curve A and shows the gradient, G_y, as a function of log E. Now any line drawn parallel to the log E axis cuts the derivative curve at equal ordinate (gradient) values. Such a line is illustrated in the figure between the points s' and r'. The length of this line is equal to the exposure scale, ES_y, for a part of the characteristic curve limited by equal gradient values. By finding the length of a series of such lines and plotting these as abscissa values against the gradient value used in each case the curve C is obtained. The line zy is equal in length to the line $s'r'$ and the same relation applies to every possible horizontal line drawn between the x-axis and the gradient value equal to the maximum gradient, γ_y, of the material. This curve, C, shows the relation between exposure scale, ES_y, and the *limiting gradient*, θ, when the gradient dD/d log E is the same at the extremities of the utilized exposure scale.

The short vertical line g is drawn at the log E value corresponding to the minimum density derived directly from measurements on the optimal positive, while the line h designates the log E value obtained by locating on the curve A the maximum density

derived from the same source. The distance between these lines, *g* and *h*, gives the exposure scale of the optimal positive as derived from measurements made on the positives. The short vertical lines *e* and *f* are so placed that the distance between them is equal to the exposure scale of the optimal positive as derived from density measurements made on the optimal negatives, the interval between *e* and *f* being symmetrically placed with respect to the interval between *g* and *h*. The mean of these two values gives the most probable value of the exposure scale of the optimal positives as shown in Table XXIX. The vertical line drawn so as to bisect the interval between *h* and *f* cuts the characteristic curve at the point *m* and the first derivative curve (curve *B*) at the point *n*. The coördinates of the point *m* give the maximum density (D_p max.) and the maximum exposure (log E_p max.) for the optical positives. The ordinate value of the point *n* gives the *limiting gradient*, θ, of the optimal positive at the high density end (shadow region) of the scale. This particular value of gradient will be designated as $s\theta_p$, denoting the limiting gradient at the *shadow* end of the utilized exposure scale. The vertical line bisecting the interval between points *e* and *g* establishes the points *p* and *o* on the characteristic and first derivative curves, respectively. The coördinates of the point *p* give the minimum density and the minimum log *E* for the optimal positive, and the ordinate value of the point *o* gives the limiting gradient at the low density (high-light) end of the utilized portion of the characteristic curve. This particular value of gradient will be designated as $h\theta_p$, denoting the limiting gradient at the *high-light* end of the utilized exposure scale. The horizontal distance between the two lines *mn* and *op* is the most probable exposure scale of the optimal positive.

Thus far in this section three new terms of sensitometric type have been introduced, and before proceeding further, it may be well for the sake of emphasis to repeat the definitions of these terms.

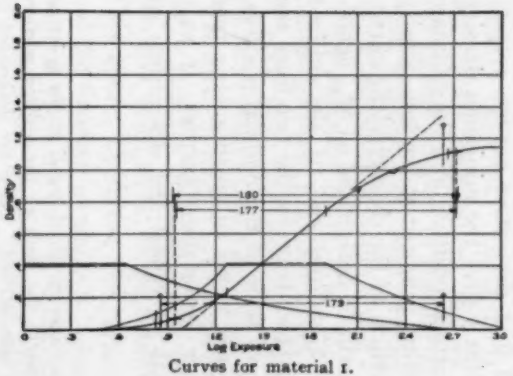
θ = limiting gradient.

Limiting gradient is the particular value of dD/d log E at the point on the characteristic curve which marks the limit of the utilized portion.

Three particular values of θ have been defined. These are designated by prefixing to the general symbol a lower case letter as follows:

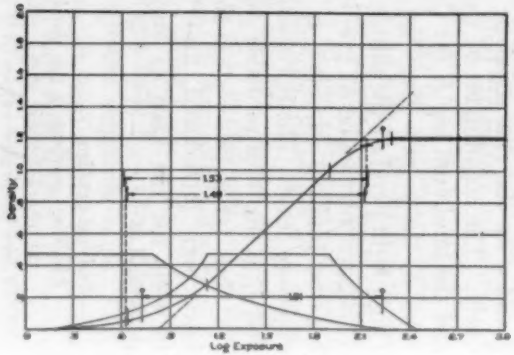
h is used to indicate the *high-light* region, that is the region of low positive density or high negative density.

FIG. 32.



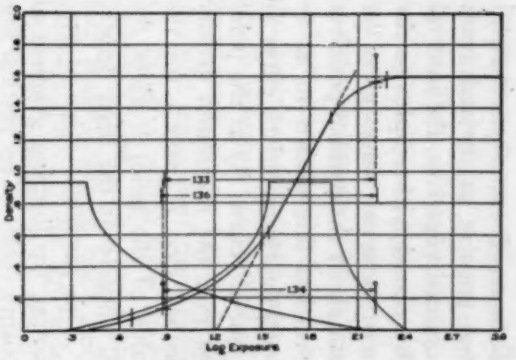
Curves for material 1.

FIG. 33.



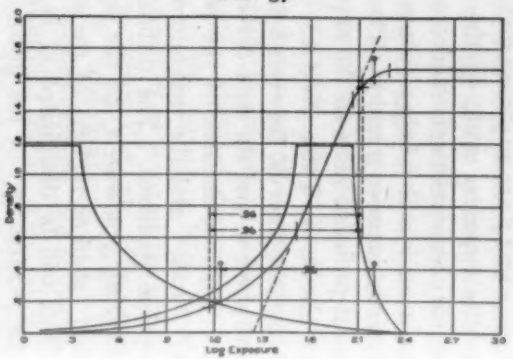
Curves for material 23.

FIG. 36.



Curves for material 9.

FIG. 37.

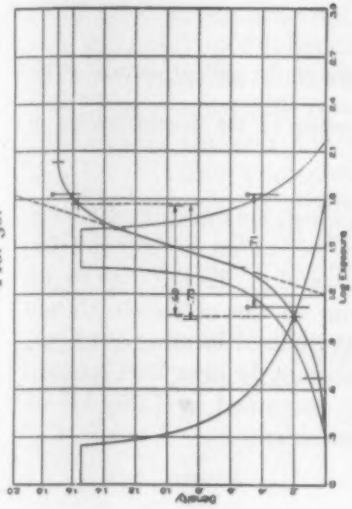


Curves for material 10.

FIG. 34.

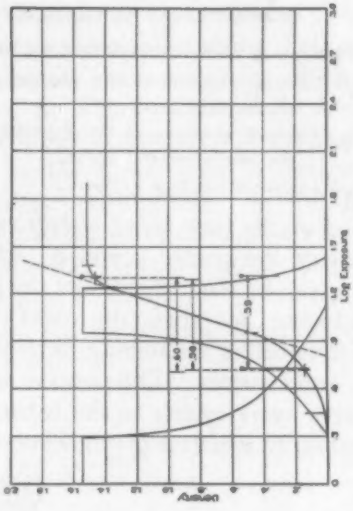
FIG. 38.

FIG. 38.



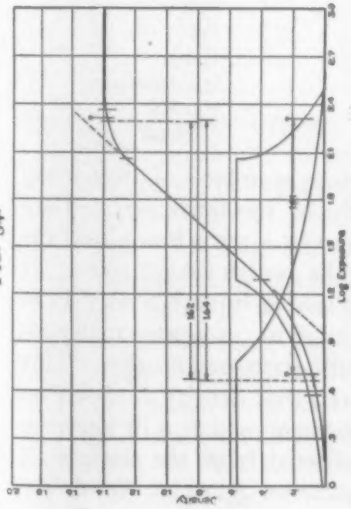
Curves for material 11.

FIG. 39.



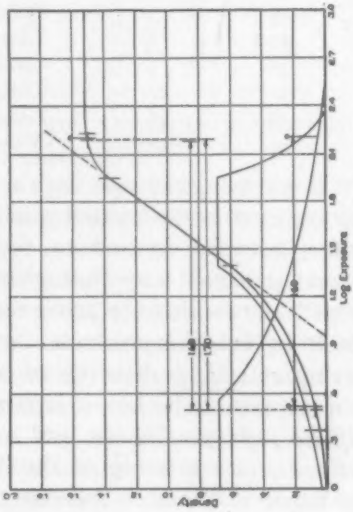
Curves for material 21.

FIG. 34.



Curves for material 4.

FIG. 35.



Curves for material 8.

FIGS. 32 to 39 inclusive. The characteristic density-log *E* curves and curves derived therefrom as illustrated by the key diagram in Fig. 31.

s is used to indicate the *shadow* region, that is the region of high positive density or low negative density.

e is used to designate *equalized* limiting gradient values, that is when the limiting gradient in the shadow region is equal to the limiting gradient in the high-light region.

Summarizing these new concepts, we have:

$s\theta_p$ = limiting gradient at the *shadow* extremity of the utilized portion of the characteristic curve.

$h\theta_p$ = limiting gradient at the *high-light* extremity of the utilized portion of the characteristic curve.

$e\theta_p$ = *equalized* limiting gradient.

For the time being attention will be concentrated on the location of the points *n* and *o* and upon the values of $s\theta_p$ and $h\theta_p$ as given by the ordinates of the points *n* and *o*. In Figs. 32 to 39, inclusive, are given the curves relating to the materials chosen as illustrating the relation between magnitude of limiting gradient, θ_p, and contrast. The positive materials may be described qualitatively by referring to the information contained in Table IX of section B, which is given below for convenience.

Fig. No.	Material No.	Surface.	Contrast.
32	1	Matte	Soft
33	23	Smooth-matte	Soft
34	4	Semi-matte	Soft
35	8	Glossy	Soft
36	9	Glossy	Hard
37	10	Glossy	Hard medium
38	11	Glossy	Hard X
39	21	Velvet	Contrast

It will be noted that for the first four members of this group the contrast is designated qualitatively by the term *soft*. They differ, however, in surface, the first being a very low gloss, the second and third a medium gloss, and the fourth a high gloss. It is well known that the same emulsion coated in such a way as to result in finished products varying in gloss produces materials having maximum densities of widely different magnitudes. This is to be expected when it is remembered that density is based on *diffuse reflection* factor and any treatment which will decrease diffusion or scattering of the light reflected from the surface of the paper will tend to increase the maximum density. While the first four materials shown in the above tabulation are all designated as *soft*, it is commonly recognized that the more glossy materials have a higher effective contrast. Materials 1, 4, and 8

are made with emulsions similar in type and hence represent a progressive increase in contrast values which is due largely to an increase in the glossiness of the surface. Material 23 (Fig. 33) represents an emulsion radically different in type and was chosen for purposes of illustration because it falls about half-way between materials 1 and 4 in contrast characteristics. Each of the positive materials represented by Figs. 35 to 38, inclusive, have glossy surfaces and do not differ appreciably in maximum black. As shown qualitatively by the terms used to designate contrast, this characteristic increases progressively. In this case the increase in contrast is due to the type of emulsion used in the manufacture of the material. Material represented by Fig. 39

TABLE XXX.

Data Illustrating the Variation in Values of Limiting Gradients, $s\theta_p$ and $h\theta_p$, with Contrast of the Material.

Positive Material.	D_y max.	γ_y.	ϕ_y.	DS_y.	ES_y.	γ_n.	$s\theta_p$.	$h\theta_p$.
1	1.15	.83	.58	1.07	1.85	1.34	.16	.30
23	1.21	.94	.70	1.15	1.65	1.11	.34	.20
4	1.40	1.13	.75	1.34	1.79	1.18	.30	.24
8	1.52	1.34	.83	1.47	1.87	1.24	.30	.31
9	1.60	1.87	.95	1.52	1.61	.94	.40	.46
10	1.67	2.37	1.05	1.61	1.54	.66	1.00	.49
11	1.70	3.12	1.23	1.67	1.36	.46	1.02	.64
21	1.52	3.21	1.60	1.49	.93	.40	.82	.76

has a semi-gloss (velvet) surface and extremely high contrast which is due chiefly to emulsion characteristics.

This group of eight papers has been chosen for use in demonstrating the way in which the magnitude of limiting gradient increases with contrast, and since it will also be used later to demonstrate the way in which final evaluation of contrast is accomplished, it may be well to summarize the sensitometric characteristics of the group so that they may be more easily compared with each other. In Table XXX values of D_y max., γ_y, ϕ_y (mean slope between the points at which gradient is equal to .2), density scale, exposure scale, and the gamma of the optimal negative (γ_n) are tabulated. It will be noted that the values of D max., γ, and ϕ increase progressively. The same is true of density scale with the exception of material 21. Exposure scale shows a general tendency to decrease, but there are some exceptions to this rule. The values do indicate, however, at least from a qualitative esti-

mate based upon their probable influence upon contrast, that the contrast of this group of eight papers should increase progressively. As will be shown later, this is actually the case. Now it will be noted in Fig. 32 that the ordinates of the points *o* and *n* are relatively small, reading .16 and .30, respectively. For this material, therefore, $s\theta_p = .16$ and $h\theta_p = .30$. Now for material 4 (Fig. 34), the values found are $s\theta_p = .30$ and $h\theta_p = .24$, thus showing an appreciable increase over the same value for material 1. In the case of material 8 (Fig. 35), a further increase in the values of limiting gradient will be noted: $s\theta_p = .30$, $h\theta_p = .31$. Material 9 shows a still further increase which, although not large, is quite definite; $s\theta_p = .40$, $h\theta_p = .46$. In the case of material 10 (Fig. 37), a very marked increase is found; $s\theta_p = 1.00$, $h\theta_p = .49$. Material 11 (Fig. 38) is most contrasty of this group and gives still higher values; $s\theta_p = 1.02$, $h\theta_p = .64$.

It is evident thus far that as the contrast of the materials increases, a greater and greater percentage of the total available density scale becomes useless. This same effect is noted throughout the entire group of materials used in this work and the evidence is so strong that the author feels little doubt as to the validity of the conclusion. In the case of material 21 (Fig. 39), the values of limiting gradient are $s\theta_p = .82$ and $h\theta_p = .76$. While the value at the shadow end of the scale is somewhat lower than in the case of No. 11; the higher value in the high-light region more than compensates for this and the *equalized* limiting gradient is higher for this material than for any other. There seems to be little doubt that the gradient at the extremes of that portion of the characteristic curve which is utilizable for the reproduction of an object of fixed brightness scale depends upon the contrast of the positive material. The materials shown in Figs. 32 to 39 strikingly demonstrate the existence of this effect. In Table XXXI are given the values of $h\theta_p$ and $s\theta_p$ for all the materials, and a careful analysis of these data shows the general tendency of the limiting gradients to increase for the materials of higher contrast.

III. EQUALIZATION OF LIMITING GRADIENTS.

The relation of the magnitude of the limiting gradient, $h\theta_p$, at the high-light end of the utilized portion of the curve to that at the shadow extremity, $s\theta_p$, is of considerable importance. These values, together with the gradients at corresponding points on the negative, give the limiting gradients of the reproduction curve

TABLE XXXI.

Values of the Limiting Gradients for the Positives Made on the Entire Group of Positive Materials Studied.

Positive Material.		Optimal Positives.				
No.	D_y max.	D_p max.	$s\theta_p$.	D_p min.	$h\theta_p$.	a.
1	1.15	1.12	.16	.06	.30	.53
2	1.19	1.16	.26	.09	.28	.93
3	1.22	1.17	.43	.11	.49	.88
4	1.40	1.38	.30	.07	.24	1.25
5	1.40	1.36	.37	.10	.32	1.15
6	1.48	1.39	1.20	.12	.38	3.16
7	1.41	1.35	.50	.16	.58	.86
8	1.52	1.50	.30	.07	.31	.97
9	1.60	1.57	.40	.13	.46	.87
10	1.67	1.56	1.00	.16	.49	2.04
11	1.70	1.57	1.02	.20	.64	1.59
12	1.66	1.58	.76	.14	.38	1.87
13	1.70	1.49	.95	.16	.60	1.58
14	1.23	1.17	.50	.04	.25	2.00
15	1.21	1.12	.60	.08	.39	1.54
16	1.50	1.43	.42	.08	.40	1.05
17	1.40	1.35	.65	.11	.56	1.16
18	1.55	1.50	.56	.08	.40	1.40
19	1.55	1.48	.78	.13	.72	1.08
20	1.16	1.14	.27	.05	.29	.93
21	1.52	1.47	.82	.14	.76	1.07
22	1.50	1.46	.36	.07	.34	1.06
23	1.21	1.16	.44	.04	.20	2.20
24	1.22	1.18	.32	.08	.25	1.28
25	1.23	1.19	.56	.07	.24	2.33
26	1.14	1.12	.20	.04	.30	.67
27	1.47	1.40	.60	.10	.29	2.07
28	1.52	1.46	.62	.14	.42	1.48
29	1.55	1.48	.95	.18	.50	1.90
30	1.28	1.25	.20	.12	.40	.50
31	1.36	1.33	.25	.11	.38	.66
32	1.40	1.38	.31	.12	.38	.82
33	1.16	1.14	.24	.04	.19	1.26
34	1.52	1.45	.58	.08	.28	2.23
35	1.28	1.26	.21	.08	.26	.81
36	1.57	1.49	.60	.11	.32	1.88
37	1.19	1.15	.22	.07	.30	.73
Mean						1.35

in the extreme high-light and shadow regions. Hence they are a direct measure of the efficiency with which brightness differences, ΔB_0, in the extreme high light and shadows of the object are reproduced. An examination of the values of limiting gradient given in Table XXXI shows that in some cases $s\theta_p$ is greater than $h\theta_p$, while in others the reverse is true. In the column designated as a (Table XXXI) are the values of the ratio, $a = s\theta_p/h\theta_p$.

Since only the straight line portion of the characteristic curve of the negative material was utilized in rendering the brightness scale of the object (the slight deviation from this condition for the negatives of higher contrast will be mentioned later), the value of gradient for each negative is a constant. It has been shown previously in the treatment of the theory of tone reproduction (see section A) that the following relationship exists between the gradient at corresponding points on the reproduction, r, the *negative material*, x, and the *positive material*, y, curves:

$$G_x \cdot G_y = G_r \tag{1}$$

where the symbol G designates gradient at the point under consideration. It follows from this that,

$$h\theta_p \cdot h\theta_n = h\theta_r \tag{2}$$

$$s\theta_p \cdot h\theta_n = s\theta_r \tag{3}$$

Reference to Table VI will show that the mean gradients, ϕ_n, of the negatives were practically the same as the gamma values. The difference between ϕ and γ is less than 5 per cent. for all negatives with the exception of No. 9. Negative No. 9 was used only in one or two cases. The assumption that the gradient of each negative was constant over its entire density scale therefore appears perfectly justifiable, hence we may write

$$h\theta_n = s\theta_n = \gamma_n \tag{4}$$

Now by combining equations (2) and (3) and substituting γ_n according to equation (4), the following relation is obtained:

$$\frac{s\theta_p \cdot \gamma_n}{h\theta_p \cdot \gamma_n} = \frac{s\theta_r}{h\theta_r} = \frac{s\theta_p}{h\theta_p} = a \tag{5}$$

The values of a in Table XXXI therefore give directly the ratio of the limiting gradients on the reproduction curve.

Goldberg, in his excellent treatment of this subject,[27] states that in the extreme high-light region two object areas differing

in brightness by 10 per cent. ($B_1/B_2 = 1.10$, that is $\Delta \log B_0 - \log B_1 - \log B_2 = .04$) should be rendered in the positive by a just perceptible difference in reflection factor and hence by a just perceptible brightness difference. He assumes that this just perceptible difference (the just noticeable difference, J.N.D. of the psychologist) is 1 per cent., $\Delta D = .004$, which would give a value for limiting high-light gradient,

$$h\theta_r = .004/.04 = .10 .$$

He states also that in the extreme shadow region a 25 per cent. difference in brightness, $\Delta \log B_0 = .10$, must be rendered as a just perceptible density difference in the positive, which gives

$$s\theta_r = .004/.10 = .04 .$$

Therefore $$a = .04/.10 = .40 .$$

The absolute values of these factors, as well as the ratio, are given considerable emphasis by Goldberg, and this relation between the limiting gradients in the high-light and shadow regions is stated as one having rather general applicability. This value of a is not in good agreement with those shown in Table XXXI, which are based upon the judgment of a large number of well-qualified observers. They apply to an object of average brightness characteristics and to a group of positive materials varying widely in sensitometric characteristics. The smallest value obtained, .50, in this work is that for material No. 30; while the mean for the entire group is 1.35, thus indicating that on the average a greater reproduction gradient is required in the extreme shadow region than in the extreme high lights. This result is the opposite to the conclusions reached by Goldberg. Of the thirty-seven values of a shown in Table XXX, thirteen are greater than unity and twenty-four are less than unity. A careful comparison of the values of a with the sensitometric constants, such as exposure scale, density scale, maximum density, gamma, mean slope, and contrast shows no appreciable correlation. While it seems reasonable to expect that various values of limiting gradient will be required for rendition of objects differing widely in brightness scale and in the distribution of maximum contrast on the brightness scale, it does not seem reasonable to suppose that with a fixed object there should be a very marked difference in values of limiting reproduction gradient for various materials. It will be well,

therefore, to examine carefully the conditions which determine the value of limiting gradient.

It will be recalled (see page 490) that three prints were made from each negative, the printing exposure being varied by an amount sufficient to produce a slightly different average density. These prints were referred to as the *light, normal,* and *dark* positives. The variation in printing exposure required to give this density variation depended to some extent on the character of the positive material, being greater for those of low contrast and less for those of high contrast. The printing exposure for the light positive was on the average 15 per cent. less than for the normal positive, while the exposure for the dark positive was approximately 15 per cent. greater. Now the *normality* of the positive density was determined by the opinion of two individuals who made the prints and is therefore dependent upon their judgment and personal taste.

Unfortunately when the judgments of print quality were made by the large group of observers, no definite record was kept with respect to their opinion regarding the correctness of exposure used in making the *normal* prints. In many cases, however, the observers expressed a preference for the *light* or *dark* print from a given negative.

When a negative having a definite density scale is printed upon any particular positive material, the ratio of the gradients at the extremities of the utilized portion of the characteristic curve depends directly upon the magnitude of the printing exposure. This is evident from Fig. 31. Thus, for example, if a negative has a density scale such that it occupies the portion of the characteristic curve between the points *p* and *m* and the exposure is such that the highest negative density is rendered by a positive density equivalent to that of point *m,* then the limiting gradients will be represented by the ordinates of the points *o* and *n* on the first derivative curve, *B*. It will be noted in this case that the ordinate of the point *n* is appreciably greater than that of point *o* and the limiting gradient in the shadow region is therefore appreciably greater than the limiting gradient at the high-light extremity. Now, if the printing exposure be increased, the highest negative density must be rendered by a greater density in the positive than in the previous case, for instance, by a density as indicated by the point *s* on curve *A*. Since we are printing the

same negative having a fixed density scale, the lowest negative density must also be rendered by a somewhat higher density in the positive as indicated by the point r. The point r is located by making the log E interval points s and r equal to that between points p and m. Now if we locate the points on the first derivative curve corresponding to these points (s and r), the points s' and r' are obtained. It will be noted that the ordinate values of s' and r' are now equal and the limiting gradient in the high-light region is equal to that in the shadow region. The ratio of the limiting gradients is therefore vitally dependent upon the magnitude of the printing exposure. Since the values of a in Table XXXI give no conclusive evidence that the ratio of limiting gradients is other than unity, it is of interest to determine how great a difference of printing exposure would be required to make the values of a unity in all cases. The limiting gradients were therefore equalized for each of the thirty-seven positive materials. In Fig. 31 the magnitude of the exposure increment required to make the limiting gradients equal is indicated by the interval ΔD log E.

The method of finding the value of $e\theta_p$ (the equalized limiting gradient) for a specified value of positive exposure scale, ES_p, is illustrated graphically in Fig. 31. The magnitude of the ES_p value in which we are interested is of course that of the mean positive exposure scale as derived from the measurements made on the optimal negatives and optimal positives, these values being tabulated for each of the positive materials in the next to the last column of Table XXIX. To find the value of $e\theta_p$ and also the exposure increment required to produce the desired equalization, it is only necessary to plot curve C (Fig. 31), which represents the exposure scale of the positive material, ES_y, as a function of equalized limiting gradient, $e\theta_y$. The method of constructing this curve is as follows: Let a horizontal line parallel to the log E axis be drawn, such as the line zr'. The exposure scale for that particular value of $e\theta_y$ (which in this case is .4) is given by the length of the line between s' and r'. Now by making the line zy equal in magnitude to $s'r'$, the point y is established. This is done for all values of $e\theta_y$ less than γ_y. By connecting these points and completing the curve by drawing a horizontal line at the ordinate value of $e\theta_y$ equal to γ_y, the curve C is constructed.

Now in order to determine the equalized limiting gradient

$e\theta_p$ for any optimal positive, it is only necessary to locate on the log E axis the value of mean exposure scale for that optimal positive and to read the value of gradient on the curve C. Thus, if the value of ES_p is 1.60, this being indicated by point x, a perpendicular drawn through x locates y at its intersection with curve C. The ordinate value of y is .4 and hence for this particular case $e\theta_p$ is equal to .4. In this way the equalized limiting gradient values as shown in the first column of Table XXXII were determined. The horizontal line drawn through the point y, established as just described, locates the points s' and r' on the curve B and the perpendiculars drawn through these points locate the limits of the characteristic curve at which the gradient values are equal. These points are indicated in Figs. 32 to 39, inclusive, by the vertical lines terminating at the top in small circles. We wish also to determine the magnitude of the exposure increment or decrement, Δ log E, required to produce equalization of the limiting gradients. To do this it is only necessary to locate on the characteristic curve (curve A) the points p and m which have ordinate values equal to the minimum and maximum density values applying to the optimal positives, these values being taken from Table XXXI. The interval between the vertical lines drawn through these points is now the mean exposure scale of the optimal positives as shown in Table XXIX. The required exposure increment or decrement is now given by the magnitude of the interval Δ log E, as shown in the figure.

It should be noted that the values of equalized limiting gradient are not in most cases the arithmetical mean of $h\theta_p$ and $s\theta_p$ values shown in Table XXXI. This, of course, results from the difference in the gradient of the shadow and high-light portions of the characteristic curve. The magnitude of Δ log E, this being the exposure increment or decrement required to produce equalization of limiting gradient, is given for each of the materials in the second column of Table XXXII, and in the third column this log E interval is expressed as a percentage of the actual exposure value. It will be noted in most cases that this correction in the exposure of the normal print is relatively small, the maximum being 22 per cent. and the average (computed without regard to the algebraic sign of the exposure correction) is only 10 per cent. If this average be computed, taking into account the algebraic sign of the correction, the value obtained is + 4 per cent. Now

TABLE XXXII.

Data Relative to the Equalization of Limiting Gradient and Sensitometric Characteristics of the Optimal Positives after Correction for Equalization of Limiting Gradient.

Paper No.	Equalized Limiting G_p.				Characteristics of Op. Pos.				
	$e\theta_p$.	$\Delta \log E$.	ΔE,%.	ΔD_p min.	D_p max.	D_p min.	DS_p.	ES_p.	ϕ_p.
1	.21	−.09	18.7	.02	1.10	.04	1.06	1.80	.59
2	.27	−.01	2.3	.00	1.16	.09	1.05	1.52	.69
3	.48	−.02	4.5	.00	1.16	.11	1.05	.83	1.26
4	.27	+.04	9.7	.01	1.38	.08	1.30	1.61	.81
5	.34	+.03	6.7	.01	1.37	.11	1.27	1.39	.91
6	.47	+.09	22.4	.03	1.45	.15	1.30	1.00	1.30
7	.55	−.02	4.5	.01	1.33	.15	1.18	.81	1.46
8	.30	−.01	2.3	.00	1.50	.07	1.43	1.70	.84
9	.40	−.01	2.3	.00	1.55	.13	1.42	1.34	1.06
10	.58	+.07	17.5	.03	1.62	.19	1.43	.96	1.49
11	.75	+.07	17.5	.03	1.63	.23	1.39	.71	1.96
12	.46	+.07	17.5	.02	1.62	.16	1.46	1.16	1.22
13	.76	+.06	14.8	.05	1.53	.21	1.32	.70	1.89
14	.33	+.08	20.3	.04	1.21	.08	1.13	1.31	.86
15	.46	+.05	12.2	.03	1.15	.11	1.04	.95	1.10
16	.41	+.01	2.4	.00	1.44	.08	1.36	1.29	1.05
17	.58	+.01	2.4	.01	1.36	.12	1.24	.89	1.40
18	.46	+.04	9.7	.03	1.52	.11	1.41	1.01	1.40
19	.74	+.01	2.4	.03	1.49	.16	1.33	.72	1.85
20	.28	−.01	2.3	.00	1.14	.05	1.09	1.31	.83
21	.78	+.005	1.0	.00	1.47	.14	1.33	.59	2.26
22	.35	+.01	2.4	.01	1.47	.08	1.39	1.39	1.00
23	.27	+.10	25.0	.03	1.20	.07	1.13	1.51	.75
24	.27	+.02	4.7	.02	1.20	.06	1.14	1.54	.74
25	.29	+.06	14.9	.02	1.22	.09	1.13	1.41	.80
26	.25	−.05	8.7	.02	1.11	.02	1.09	1.52	.72
27	.37	+.06	14.8	.01	1.43	.11	1.32	1.22	1.08
28	.46	+.03	7.2	.02	1.48	.16	1.32	1.15	1.15
29	.59	+.05	12.2	.03	1.51	.21	1.30	.82	1.59
30	.30	−.10	20.6	.03	1.22	.09	1.13	1.49	.76
31	.34	−.05	8.7	.02	1.31	.09	1.22	1.43	.85
32	.33	−.08	16.8	.02	1.36	.10	1.26	1.40	.90
33	.22	+.02	4.7	.01	1.14	.05	1.09	1.68	.64
34	.32	+.06	14.9	.02	1.50	.10	1.40	1.50	.93
35	.24	−.04	8.8	.02	1.24	.06	1.18	1.71	.69
36	.38	+.06	14.0	.03	1.53	.14	1.39	1.32	1.05
37	.26	−.06	12.9	.01	1.14	.06	1.08	1.51	.72
Mean			10.4	.011					

in order to produce a just perceptibly lighter or darker print, it is necessary to increase or decrease the exposure by approximately 15 per cent. It will be seen, therefore, that the average correction in printing time required to produce the condition of equal limiting gradients is appreciably less than that required to produce a perceptible difference in print density. Perhaps the smallness of these corrections may be most emphatically illustrated by expressing the increment or decrement of maximum print density resulting from the Δ log E values shown in Table XXXII. These density corrections, ΔD_p min., are shown in the fourth column. The maximum increment in D_p max. was found to be .06, while the maximum increment of D_p min. was only .04, the average being .012 and .011, respectively. This demonstrates that the required change in printing exposure to produce equality of limiting gradients is of almost negligible order. Furthermore, a reëxamination of those cases in which the Δ log E values were relatively large, such as in the case of materials Nos. 6, 10, 11, 12, 13, 14, 32, etc., was made by a few well-qualified observers and in every case the average opinion indicated that a more correctly exposed print would have been obtained by making a correction to the printing exposure of *normal* print of the sign and approximate magnitude indicated by the values of Δ log E (Table XXXII). This point has been given most careful consideration. The author feels that the equalization of the limiting gradients is not only justified, but actually produces better quality in the optimal prints. Now by reading the coördinates of the points corresponding to s and r (Fig. 31), it is possible to determine the D_p max., D_p min., DS_p, ES_p, and θ_p for the optimal prints corrected to give equalized limiting gradients. These values are tabulated in the columns designated in Table XXXII. A comparison of these values with those in Tables XXIX and XXXI will show that the modification of these values pertaining to the optimal positives are in all cases relatively small, and that this correction of printing exposure to produce equalized limiting gradient does not to any extent change the numerical values applying to the optimal positives. In a few cases it changes the minimum and maximum density values by .04. The exposure scale and density scale values are practically unaltered and no appreciable changes in the values of mean gradient, θ_p, result. For an object such as is used in this work, the author feels there

is very strong evidence that *optimal* quality is obtained when limiting gradient is equal in the extremes of the brightness scale.

The values of $e\theta_p$ (Table XXXII) show conclusively that for a fixed value of BS_0 the optimal positive is not limited by points having the same gradient in the case of papers varying in contrast. Hence, if a series of contrast values is to be computed which will be relatively correct when applied to the rendition of an object of fixed brightness scale, it will be necessary to base the evaluation of contrast upon portions of the characteristic curve limited by an equalized gradient value which must depend upon the characteristics of the positive itself. It is proposed now to compute contrast for the various positive materials used as based upon their ability to reproduce the test object (Willow Pond) chosen for this work. The portions of the characteristic curves used will therefore be limited by the points analogous to *s* and *r* in the key diagram (Fig. 31).

IV. EVALUATION OF THE CONTRAST FUNCTION.

From a consideration of the fundamental psychophysical laws the conclusion has been drawn that the contrast concept as applied to photographic papers consists of two factors or attributes, namely, *extent* and *rate* and in the discussion of this subject given in the introductory section of this paper (section A) an evaluation of contrast in general terms was made.

$$\Omega \text{ (contrast)} = c_3 (\varepsilon \cdot \tau) = c_3 (D \text{ max.} - D \text{ min.}) \cdot (dD/d \log E)$$

where ε (extent) $= c_1 (D \text{ max.} - D \text{ min.})$

and τ (rate) $= c_2 (dD/d \log E)$.

It now becomes necessary to develop this rather general evaluation of contrast into a form applicable to the numerical values relative to the materials with which we are dealing.

Extent Factor.—We will consider first the evaluation of the extent attribute of contrast. There seems to be little doubt as to the validity of the psychophysical law which states that the magnitude of the sensation produced is *directly proportional* to the logarithm of the stimulus values. It is recognized of course that this relationship does not hold in the region of extremely low and high stimulus values, however, for brightness levels greater than 5 millilamberts and less than 1000 millilamberts direct proportionality between the stimulus and sensation exists. The brightness level at which photographic prints are usually viewed lies

within this range and we shall base our evaluation of contrast on the assumption that this condition is fulfilled. Density is a logarithmic form of expressing brightness and hence the contrast sensation resulting from a density difference is directly proportional to the magnitude of that difference. The numerical value of density scale ($DS_y = D_y$ max. $- D_y$ min., or in case of a positive the value of D_p max. $- D_p$ min.) expresses relatively the magnitude of the extent factor (ε).

The value of the constant of proportionality, c_1, must depend upon the magnitude of the least perceptible difference in brightness (just noticeable difference, J.N.D.). It is customary in psychophysical work to adopt the increment of stimulus which causes a just perceptible increment in the resulting sensation as a unit in which to express sensation magnitudes. While such procedure may be open to criticism from the metaphysical standpoint, it seems to be the only course open for the expression of sensation in numerical terms. The adoption of the least perceptible difference as the unit of sensation therefore seems permissible and as a matter of fact for practical purposes is useful. In the present instance we shall regard the least perceptible difference in brightness as the satisfactory unit in which to express the sensation of contrast and for convenience will denote this factor by the symbol $l\triangle D$. If it is desirable, therefore, to express the *extent* factor of contrast in absolute units, it is only necessary to divide the density difference D max. $- D$ min. by the value of $l\triangle D$. The most reliable work on this subject indicates that two fields differing in brightness by 1.7 per cent. are just perceptibly different in brilliance. Expressing this brightness ratio in terms of density, we obtain

$$l\triangle D = .007.$$

It is obvious that dividing a density difference by $l\triangle D$ gives the actual number of just perceptible sensation steps in that density interval, and it seems entirely logical to assume that this number constitutes a numerical expression of the extent factor of contrast in absolute terms. We may, therefore, complete the evaluation of the extent factor by writing

$$\varepsilon = \frac{1}{l\triangle D} (D \text{ max.} - D \text{ min}).$$

where $l\triangle D = .007$.

The fact that we have assumed a constant value for $l\triangle D$ should

not be construed as indicating that this general method of evaluating contrast depends essentially on the permissibility of making such an assumption. The relation between stimulus (brightness) and sensation (brilliance) is known with considerable certainty,[10, 11, 12, 13] hence the number of just perceptible sensation steps between any specified brightness limits can be determined. As a matter of fact, it is necessary in work on tone reproduction to take account of the variation in the value of $l\triangle D$ and a method for doing this has already been described.[26] The assumption of constancy in $l\triangle D$, however, simplifies procedure and is entirely justified in the computation of contrast since it is not only desirable, but essential, that the contrast values obtained for the various materials shall apply to these materials when used under precisely the same conditions from the standpoint of illumination level.

Rate Factor.—This factor has the general form

$$r = c_3 \, (dD/d \log E).$$

The value of $dD/d \log E$, of course, refers only to a single point on the characteristic curve, and since we are in general concerned with a finite portion of the curve, it follows that we must evaluate $dD/d \log E$ for that portion of the characteristic curve which is utilized. Thus $dD/d \log E$ must be evaluated for the entire characteristic, or for that portion of the characteristic utilized in making the print. In case only the straight line portion is used, for which $dD/d \log E$ is constant, the evaluation of this is relatively simple and it is only necessary in such a case to use the actual value of the gradient as a direct measure of the magnitude of the *rate* attribute of contrast. In general, however, the value of the gradient is variable and the method to be followed in its evaluation is not so self-evident. The most obvious course to follow would be simply to determine the mean value of $dD/d \log E$, with respect to equal increments of $\log E$, for the particular portions of the characteristic curve under consideration. Careful analysis of such procedure, however, indicates that this does not lead to a satisfactory result. When it is considered that our concept of contrast must be based fundamentally upon the visual perception of a series of areas differing in brightness or density, it seems more logical that the determination of the effective value of $dD/d \log E$ should be based upon equal density increments rather than upon equal exposure increments. In other words, $dD/d \log E$ should be

expressed as a function of density rather than of log exposure in order to satisfactorily evaluate this factor for use in the computation of contrast. This reasoning seems to have adequate support on the basis of fundamental psychophysical relations and, as will be shown later, leads to more satisfactory results from the practical standpoint than if the evaluation were made with reference to exposure.

It will be necessary, therefore, to determine the mean value of gradient on the basis of equal *density increments* as well as for equal *log exposure increments.* To indicate an average or mean value of gradient we shall place a bar above the usual symbol for this factor and indicate with respect to which variable the average refers by placing in parenthesis following the symbol for gradient the letter D to indicate density and x to indicate exposure. Thus,

$\overline{G}$ = average gradient

$\overline{G}(D)$ = average gradient evaluated with respect to equal increment of density (D).

$\overline{G}(x)$ = average gradient evaluated with respect to equal increments in log exposure (x)

$x = \log E$

The usual characteristic curve shows the relation between density and log E,

$$D = f(x).$$

The first derivative of this function gives the relation between gradient and log E

$$G = f'(x)$$

$$\overline{G}(x) = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \frac{dD}{dx}\, dx \qquad (1)$$

$$= \frac{D_2 - D_1}{x_2 - x_1} = \frac{DS}{ES} = \phi\,. \qquad (2)$$

The value of gradient $dD/d \log E$ may also be expressed as a function of density (D) as illustrated in Fig. 40, which shows graphically the form of

$$G = f_1(D)$$

$$\overline{G}(D) = \frac{1}{D_2 - D_1} \int_{D_1}^{D_2} \frac{dD}{dx}\, dD\,. \qquad (3)$$

Since density (D) is some function of x, it follows that if

$$D = D_1, \qquad x = x_1$$

and if

$$D = D_2, \qquad x = x_2.$$

Therefore

$$\overline{G}(D) = \frac{1}{D_2 - D_1} \int_{x=x_1}^{x=x_2} \left(\frac{dD}{dx}\right) \frac{dD}{dx}\, dx \tag{4}$$

$$\overline{G}(D) = \frac{1}{D_2 - D_1} \int_{x=x_1}^{x=x_2} \left(\frac{dD}{dx}\right)^2 dx.$$

$$\overline{G}(D) = \frac{1}{D_2 - D_1} \cdot (x_2 - x_1) \cdot \overline{G^2}(x). \tag{5}$$

From equation (2), it is evident that

$$\frac{x_2 - x_1}{D_2 - D_1} = \frac{1}{\phi} = \frac{1}{\overline{G}(x)}. \tag{6}$$

Hence

$$\overline{G}(D) \cdot \overline{G}(x) = \overline{G^2}(x) \tag{7}$$

$$\overline{G}(D) \cdot \phi = \overline{G^2}(x) \tag{8}$$

$$\overline{G}(D) = \frac{\overline{G^2}(x)}{\phi} = \overline{G^2}(x) \cdot \frac{ES}{DS}. \tag{9}$$

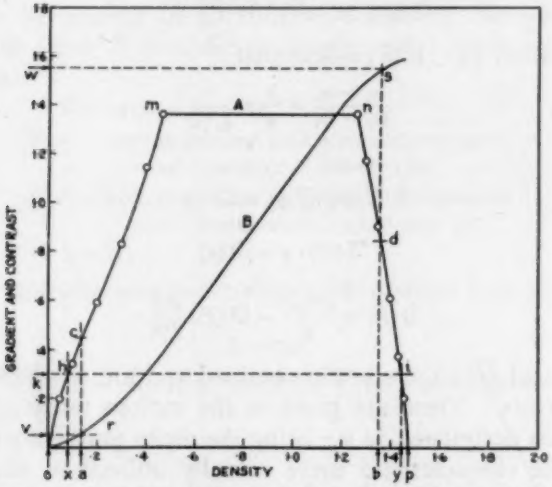
Values of $\overline{G}(x)$ are easily obtained and are available for the optimal prints. These are given in the various tables of data in the columns designated as ϕ_p, being the mean gradient of the portion of the characteristic curve actually utilized in making the optimal positives. Mean values of gradient with respect to log E are also available for the positive materials themselves. This evaluation is made for the portion of the characteristic curve included between the points where $G = .2$. These values are also given in the tables and are indicated by ϕ_y.

In order to obtain values for $\overline{G}(D)$ it is only necessary to determine the average value of $\overline{G^2}(x)$. This can be done by squaring all of the ordinates of the first derivative curve for the material under consideration. These values may then be plotted as a function of log exposure and the area enclosed by the curve determined. This area divided by the log E interval (the $x_2 - x_1$ term of the equations above) gives the desired values of $\overline{G^2}(x)$. Inserting this and the value of $\overline{G}(x)$, that is ϕ, in equation (9) determines the

desired value of *average gradient* evaluated with respect to *equal intervals of density.*

If we consider that the differential $dD/d \log E$ is merely the limiting form of the ratio between quantities of finite character, ΔD and $\Delta \log E$, it is evident that the dimensions of this term are of the same type as D max. $- D$ min., both being an expression of a ΔD value. The constant of proportionality c_2 must therefore be of the same nature as that occurring in the case of the extent factor. This idea may be made more definite by considering that the term $\Delta D/D \log E$ is merely a formal way of expressing the magnitude

FIG. 40.



Curve *A* shows the relation between gradient, $dD/d \log E$, and density. Curve *B* is the integral of curve *A* and shows directly the relation between contrast and density.

of the density increment, ΔD, per unit increment in $\log E$. If ΔD be divided by the just perceptible density difference, $l\Delta D$, the ratio

$$\frac{\Delta D / l\Delta D}{\Delta \log E} = \frac{1}{l\Delta D} \cdot \frac{\Delta D}{\Delta \log E},$$

then indicates the number of just perceptible density differences per unit exposure increment. If it is desirable, therefore, to express the *rate* factor of contrast in absolute terms, it is only necessary to introduce the numerical value of $l\Delta D$ as the proportionality constant. We may therefore write as the final evaluation of the *rate* factor

$$\tau = \frac{1}{l\Delta D} \cdot \bar{G}(D) .$$

There is no reason *a priori* to assume that the magnitude of contrast is directly proportional to the product of the two factors, *rate* and *extent*. Actual comparisons of contrast values computed by using the product of the two factors with the subjective impression of the contrast of photographic materials as judged qualitatively by their actual use in practice does, however, indicate that such procedure gives consistent results. We, therefore, at least for the purposes of this paper, will assume that contrast is given simply by the product of the two factors. Our final evaluation of contrast therefore becomes

$$\Omega = \frac{1}{l\Delta D}(D \text{ max.} - D \text{ min.}) \cdot \left[\frac{1}{l\Delta D}\bar{G}(D)\right]$$

$$\Omega = \frac{(D \text{ max.} - D \text{ min.}) \cdot [\bar{G}(D)]}{l\Delta D^2} .$$

It should be noted that contrast in absolute terms is obtained by multiplying the number of just perceptible density differences included in the density scale by the average number of the just perceptible density differences per unit difference in log E. The factor $l\Delta D$ therefore appears in the equation as a square and hence contrast is expressible in terms of the squared least perceptible density difference. Formally, contrast is analogous to an area which is expressible in form of the squared unit of length. It should be possible, therefore, to represent contrast directly as an area, and as will be shown in the following pages, such is the case.

V. GRAPHIC METHOD FOR OBTAINING CONTRAST VALUES.

The evaluation of contrast may also be accomplished graphically by plotting gradient, G, as a function of density, D. This can be done by reading from the first derivative of the $D - \log E$ curve values of gradient corresponding to the various values of density. For example, let a density value be as indicated by a point w (Fig. 31). A horizontal line through this point cuts curve A at the point k. A perpendicular through k cuts curve B at the point t, the ordinate value of which is the gradient of curve A at the density w. Obtaining a series of values in this manner and plotting them as a function of density gives the curve A in Fig. 40. This curve was plotted from the data relative to the material *Y*-27. Now if the area enclosed by the

figure *omnp* be determined and this value divided by the length of the base line, *op*, the value obtained will be, $\overline{G}(D)$, the mean or average value of G, this average being based upon equal density increments or intervals. Referring again to our contrast equation, it will be seen that contrast is proportional to the product of density scale (which in the case of the figure is given by the length of the line *op*) by the average gradient (evaluated with respect to equal density increments).

We now have determined both of these factors, and as a matter of fact the area enclosed by the figure *omnp* is the product of these two factors. Therefore the area enclosed by the curve representing gradient as a function of D gives directly the relative contrast. If it be desired to express in absolute terms, that is in terms of the sensation unit which we have adopted, the just noticeable difference, it is only necessary to multiply the area value by $(1/.007)^2$. The curve A also enables us to determine the contrast value for any particular part of the density scale. Thus if it be desired to determine the contrast of that portion lying between points designated as a and b on the density scale, it is only necessary to draw perpendiculars through these points, cutting curve A at the points c and d. The contrast available when this portion of the density scale is utilized is now given by the area enclosed in the figure *acmndb*. Also, if it be desired to evaluate contrast for any specified value of equalized limiting gradient, $e\theta_p$, it is only necessary to locate such value on the gradient scale, for example, point k, and by drawing a horizontal line through this point establish the points h and i at the intersection of this horizontal line with the curve A. The contrast available under these conditions is now given by the area enclosed by the figure *xhmniy*.

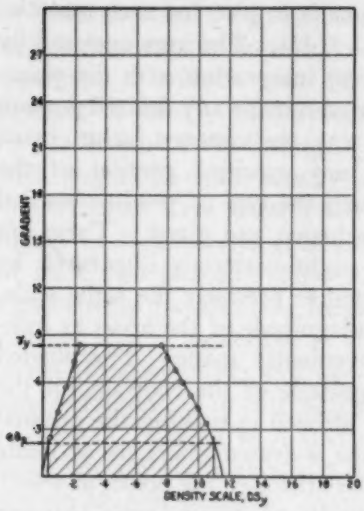
Since no satisfactory analytical equation of general applicability is available for expressing the relation $D = f(\log E)$, it is not possible to evaluate analytically the expression, $G = f_1(D)$, represented by curve A. Curve A may, however, be integrated graphically and such procedure yields curve B (Fig. 40), which shows the relation between contrast and density. Contrast values between any specified density values may now be read directly from curve B. For instance, if the density limits be as indicated by points a and b, the perpendiculars through these points establish the points r and s on curve B. Now if the ordinate values of

points s and r be v and w, respectively, contrast for this section of the density scale is given by $w - v$.

In practice the simplest procedure is to plot for each material a curve similar to A (Fig. 40), $G = f(D)$. The area enclosed by this curve can then be determined by integration with the planimeter. This integration can be carried out for any desired portion of the density scale and in this way the contrast value (area enclosed) can be determined for any specified portion of the density scale. In order to show clearly the type of results obtained by this method, Figs. 41 to 48, inclusive, are given. These are the total contrast curves for the eight materials illustrated by Figs. 32 to 39. These are all plotted to precisely the same scale, so that a direct comparison of the magnitude of the areas as indicated by the shading can be conveniently made. The shaded area in each case indicates the magnitude of the contrast for the portion of the characteristic curve utilized in making the optimal positives. The horizontal dotted line is drawn where the ordinate value is equal to $e\theta_p$ and perpendiculars through the points at which this line cuts the curve form the limits of density scale between which integration is carried out. Visual inspection of this group of eight curves shows a progressive and marked increase in contrast which is in good agreement with the qualitative evaluation of the contrast of these materials as based on the sensitometric constants applying to them (Table XXX).

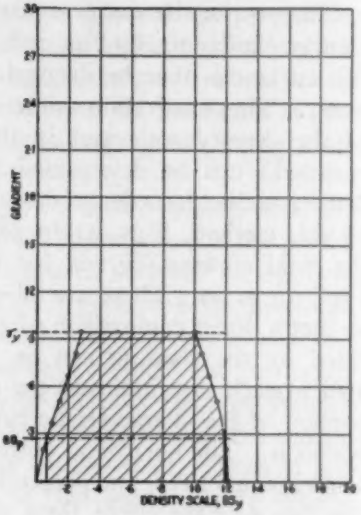
The values of contrast for this group of materials are given in Table XXXIII. In the first section the values apply to the portion of the density scale utilized in making the optimal positives, while in the second section of the table are given values of total contrast based upon the portion of the characteristic curve limited by the points of gradient equal to .2. It will be noted that the value of contrast, Ω_p, increases progressively for this group of eight materials. An inspection of the values in the column designated as ε_p and τ_p shows that in some cases the increment in contrast is due to an increment in the values of both rate and extent, while in other cases the increase is due to an increment in only one of the attributes of contrast. Thus in the case of materials 8, 9, and 10 the ε_p value is practically constant, the increase in contrast being due to a progressive increase in the value of τ_p. It should be noted also that Ω_p is very nearly equal to $\Omega_{.2}$ for the materials of low contrast, and that as contrast increases the differ-

FIG. 41.



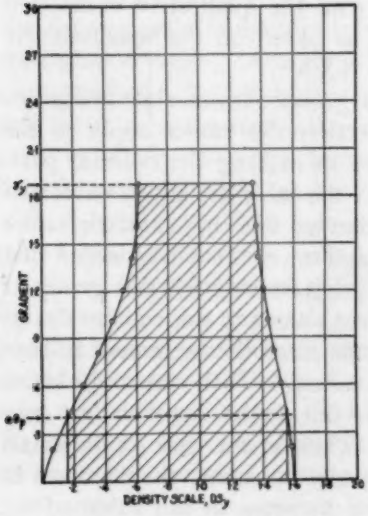
Contrast area for material 1.

FIG. 42.



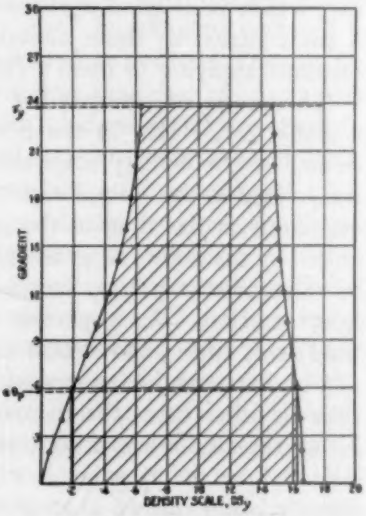
Contrast area for material 23.

FIG. 45.



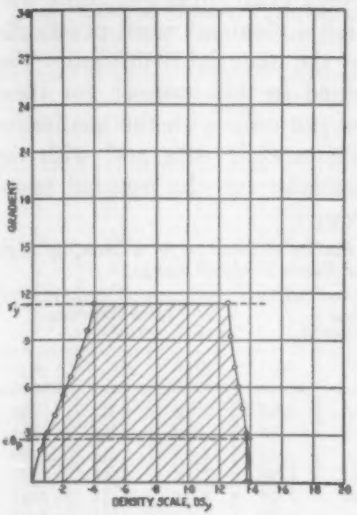
Contrast area for material 9.

FIG. 46.



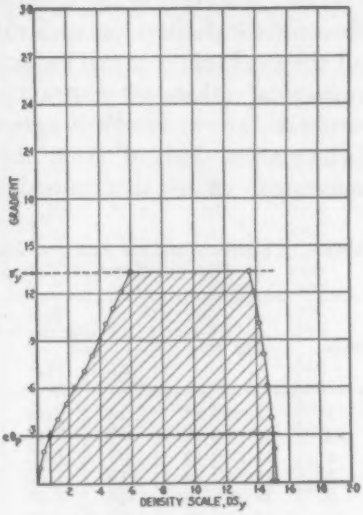
Contrast area for material 10.

FIG. 43.



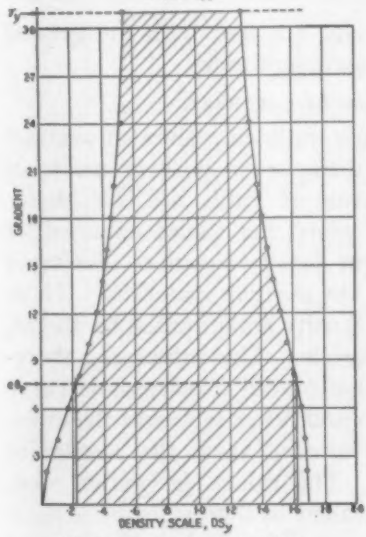
Contrast area for material 4.

FIG. 44.



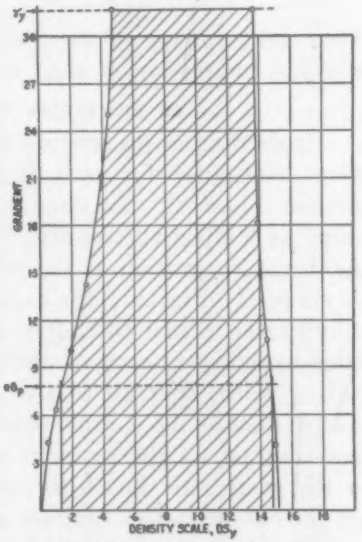
Contrast area for material 8.

FIG. 47.



Contrast area for material 11.

FIG. 48.



Contrast area for material 21.

FIGS. 41 to 48 inclusive. The figures in this group illustrate the relative magnitude of contrast for the eight materials, the areas of the shaded portions being directly proportional to contrast as computed by the method proposed.

ence between these two values becomes progressively greater. In no case, however, is the difference very great, thus indicating that the optimal positive on each material is obtained when practically all the available contrast capacity of the material is utilized. The numerical values of contrast obtained in this manner for these materials are in excellent agreement not only with the qualitative information derived from the sensitometric data and with the numerical values determined statistically for the optimal negatives and positives of these materials, but also with the general practical impression of their contrast magnitudes.

TABLE XXXIII.

Values of Contrast and the Rate and Extent Factors of Contrast for a Group of Eight Typical Materials Covering the Entire Contrast Range.

Paper.	Fig. Nos.	Partial Contrast (For Optimal Positives).				Total Contrast. $e\theta_y = .2$.		
		$e\theta_p$.	Ω_p.	ε_p.	τ_p.	$\Omega_{.2}$.	$\varepsilon_{.2}$.	$\tau_{.2}$.
1	32 and 41	.21	.72	1.06	.68	.72	1.07	.67
23	33 and 42	.27	.93	1.13	.82	.95	1.15	.83
4	34 and 43	.27	1.26	1.30	.97	1.28	1.34	.96
8	35 and 44	.30	1.55	1.43	1.08	1.58	1.47	1.07
9	36 and 45	.40	1.96	1.42	1.38	2.03	1.52	1.33
10	37 and 46	.58	2.69	1.43	1.88	2.79	1.61	1.73
11	38 and 47	.75	3.33	1.39	2.40	3.46	1.67	2.08
21	39 and 48	.78	3.60	1.33	2.71	3.69	1.49	2.48

VI. NUMERICAL VALUES OF CONTRAST.

Proceeding in the manner already outlined, values of contrast were determined for each of the thirty-seven positive materials used in this work. In the first section of Table XXXIV, designated as "effective contrast," are given the values obtained by the direct integration of the gradient-density function, illustrated by curve *A* (Fig. 40) for each of the positive materials. These values refer to that portion of the density scale limited by the $e\theta_p$ value applying to each of the materials. The values, Ω_p, therefore, give the contrast of the portion of the characteristic curve used in making the optimal positives and hence represent the effective contrast of the material as evaluated from the standpoint of the particular test-object used. The values designated as ε_p are the extent factors and are equivalent to the values of density scale for the optimal positives. The rate factor, τ_p, is obtained by dividing Ω_p by ε_p. The limiting gradient values applying in each case are shown in the column so designated.

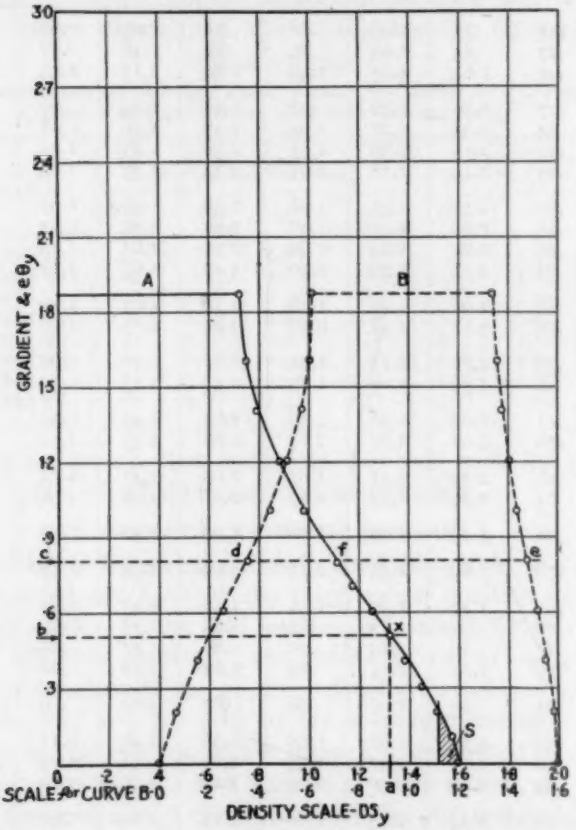
TABLE XXXIV.

Values of Contrast for the Positive Materials Studied, Computed Both on the Basis of the Equalized Limiting Gradient, $e\theta_y$, Equal to .20 and the Equalized Limiting Gradient Equal to That Observed Experimentally for the Optimal Positives on the Various Materials.

Paper.	Effective Contrast. $e\theta = e\theta_p$.				Total Contrast. $e\theta_y = .20$			$\Omega_0 - \Omega_{.2}$.
	$e\theta_p$.	Ω_p.	ϵ_p.	τ_p.	$\Omega_{.2}$.	$\epsilon_{.2}$.	$\tau_{.2}$.	
1	.21	.72	1.06	.68	.72	1.07	.67	.009
2	.27	.84	1.05	.80	.86	1.12	.77	.009
3	.48	1.64	1.05	1.56	1.69	1.17	1.44	.006
4	.27	1.26	1.30	.97	1.28	1.34	.96	.007
5	.34	1.33	1.27	1.05	1.37	1.36	1.01	.006
6	.47	2.15	1.30	1.65	2.20	1.43	1.54	.006
7	.55	2.44	1.18	2.07	2.53	1.37	1.85	.005
8	.30	1.55	1.43	1.08	1.58	1.47	1.07	.006
9	.40	1.96	1.42	1.38	2.03	1.52	1.33	.008
10	.58	2.69	1.43	1.88	2.79	1.61	1.73	.006
11	.75	3.33	1.39	2.40	3.46	1.67	2.08	.005
12	.46	2.02	1.46	1.38	2.10	1.61	1.30	.007
13	.76	3.12	1.32	2.36	3.28	1.65	1.98	.006
14	.33	1.25	1.13	1.11	1.27	1.18	1.08	.006
15	.46	1.39	1.04	1.33	1.44	1.17	1.23	.005
16	.41	1.88	1.36	1.38	1.92	1.43	1.34	.004
17	.58	2.02	1.24	1.63	2.08	1.37	1.52	.004
18	.46	2.59	1.41	1.83	2.63	1.51	1.74	.004
19	.74	2.89	1.33	2.17	2.98	1.52	1.96	.003
20	.28	1.09	1.09	1 00	1.09	1.12	.98	.005
21	.78	3.60	1.33	2.71	3.69	1.49	2.48	.003
22	.35	1.47	1.39	1.05	1.49	1.45	1.03	.006
23	.27	.93	1.13	.82	.95	1.15	.83	.006
24	.27	.86	1.14	.75	.89	1.17	.74	.005
25	.29	1.01	1.13	.89	1.03	1.17	.88	.006
26	.25	.87	1.09	.80	.87	1.10	.79	.005
27	.37	1.52	1.32	1.15	1.56	1.40	1.11	.007
28	.46	1.76	1.32	1.33	1.82	1.46	1.25	.007
29	.59	2.60	1.30	2.00	2.68	1.49	1.80	.006
30	.30	1.13	1.13	1.00	1.15	1.20	.96	.009
31	.34	1.15	1.22	.94	1.18	1.30	.91	.007
32	.33	1.30	1.26	1.03	1.32	1.33	.99	.008
33	.22	.83	1.09	.76	.83	1.09	.76	.006
34	.32	1.42	1.40	1.01	1.45	1.46	.99	.006
35	.24	1.08	1.18	.92	1.10	1.20	.92	.007
36	.38	1.60	1.39	1.15	1.64	1.49	1.10	.008
37	.26	.87	1.08	.81	.88	1.12	.79	.008
Mean								.006

In the second section of the table, designated as *total contrast,* are the values obtained by integrating over the range of density scale limited by $e\theta_y = .20$. The values of contrast and of the rate and extent factors are designated as usual. It should be noted that for the materials of low contrast there is practically no differ-

FIG. 49.



Curve *A* (derived directly from curve *B*) shows the equalized limiting gradient, $e\theta_y$, as a function of density scale, DS_y.

ence between the value of total and effective contrast. For the more contrasty materials the difference becomes greater. In other words, it is not possible in the case of the more contrasty materials to utilize fully the available contrast, at least for the reproduction of a normal landscape of the type represented by

"Willow Pond." The non-utilizable portion of contrast in no case is very great since the maximum difference is of the order of 3 per cent.

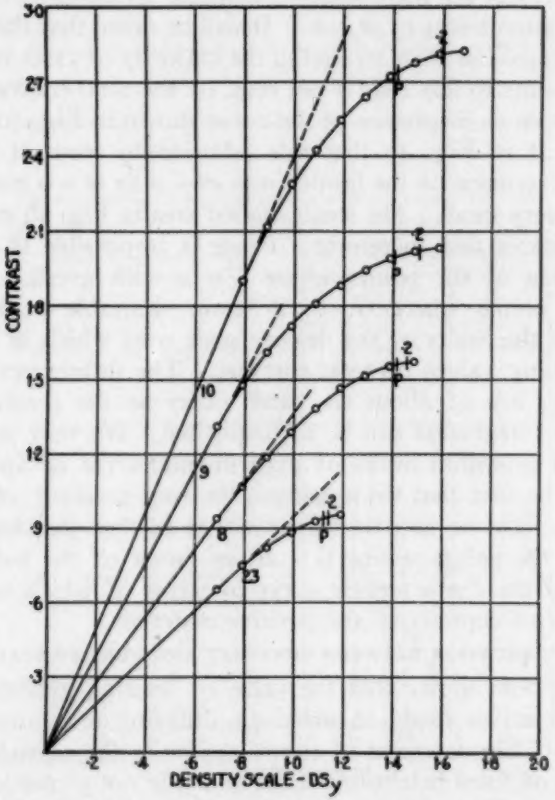
If the integration be made between the points on the characteristic curve where $G = 0$ a value, Ω_0, slightly greater than $\Omega_{.2}$ is obtained. The difference, however, is relatively small. In the last column of Table XXXIV are shown values of $\Omega_0 - \Omega_{.2}$. These represent the increments of contrast obtained by extending the integration limits to $e\theta_y = 0$. It will be noted that these values are in all cases less than .01 and in the majority of cases this increment amounts to less than 1 per cent. of the total contrast. It is evident from an inspection of the curve shown in Fig. 40 and also of curve A in Fig. 49 that this addition to contrast resulting from an extension of the limits from $e\theta = .2$ to $e\theta = 0$ must necessarily be very small. The small shaded area in Fig. 49 designated as S indicates this increment. Since it is possible to establish the position of the points where $G = .2$ with greater precision than the points where $G = 0$, it seems desirable to adopt the former as the limits of the density scale over which to integrate for obtaining values of total contrast. The differences between Ω_0 and $\Omega_{.2}$ are of about the same order as the precision with which the integration can be accomplished. We may say, therefore, that to within limits of experimental error Ω_0 and $\Omega_{.2}$ are equal. The fact that the equalized limiting gradient, $e\theta_p$ (Table XXXII), is in no case less than .20, is further justification for adopting the points where $G = .20$ as limits of the total useful portion of the characteristic curve in terms of which to express total contrast capacity of the positive materials.

VII. RELATION BETWEEN CONTRAST AND DENSITY SCALE.

It has been shown that the value of limiting gradient of the optimal positives made on materials differing in contrast is not constant. This statement of course applies to the reproduction of an object of fixed brightness scale, and it is not probable that the same values of limiting gradient would be obtained for objects of different brightness scales. It is therefore of interest to determine the relation between contrast and limiting gradient. It has also been shown that when the printing exposure is adjusted to its optimal value the limiting gradient at the shadow end of the scale, $s\theta_p$, is equal to the limiting gradient at the high-light end of the scale, $h\theta_p$. There is no reason to believe that for objects of differ-

ent brightness scale, such equality of limiting gradient will not exist. We wish, therefore, to determine for a few typical materials the relation between contrast and density scale as limited by points of equal gradient. This is most conveniently accomplished by plotting the curve showing density scale, DS_y, as a

FIG. 50.



Curves showing the relation between contrast and density scale for the materials as indicated by the figures attached to the curves.

function of the equalized limiting gradient $e\theta_y$. This is illustrated for material No. 9 by curve *A* (Fig. 49).

This is obtained by using curve *B* (shown dotted) which shows gradient, G_y, as a function of density, *D*. A horizontal line such as *cde* establishes the points *d* and *e* at its intersection

points with curve B, the length of the line de giving the density scale, DS_y, for the value of gradient as indicated by the point c. Now by making the distance cf equal to de the point f is established. Proceeding similarly for other values of gradient between o and γ_y and drawing the horizontal line at $G = \gamma_y$, the curve A is established. Now in order to obtain values of contrast for various values of density scale, it is only necessary to integrate curve A over the density scale from o to the desired value. Proceeding in this manner values were obtained from which the curves shown in Fig. 50 are plotted. These apply to the materials as indicated by the numbers attached to the various curves. It will be noted that contrast increases according to the straight line function until DS_y becomes equal to the density latitude, DL_y, of the material and then continues to increase at a lesser rate until a maximum is reached where DS_y becomes equal to D_y max. for the material. The short vertical lines drawn through each of the curves and designated as p represent the contrast value for the optimal positives made on these materials, while the vertical lines designated as .2 represent the contrast values for the scale limited by the points for which $G = .2$. The relation between contrast and utilized density scale, as shown by these curves, is in entire agreement with the practical idea relating to this matter, it being usually considered that the contrast continues to increase until the entire available density scale is utilized.

VIII. EVIDENCE AGAINST THE USE OF ϕ FOR THE EVALUATION OF CONTRAST.

As suggested previously, it may appear to some that the use of ϕ, mean gradient on a basis of equal exposure increments, should be satisfactory for the evaluation of contrast. This conclusion is not supported by an analysis of the psychophysical relations involved in our perception of contrast, nor does it lead to satisfactory results from the practical standpoint. Since the mere statement of this fact may not be convincing, it seems desirable to present some evidence in its support. Let us therefore determine the way in which values of contrast computed on this basis vary with the magnitude of the density scale utilized. We will again assume that the density scale is to be limited by points of equal gradient. Contrast computed in this way will be given by the product of ϕ multiplied by DS_y.

$$\phi = \frac{DS}{ES}.$$

Therefore $$\phi \cdot DS = \frac{DS^2}{ES}.$$

Hence it is only necessary to determine corresponding values of density scale and exposure scale for a series of values of $e\theta_y$. The required data can be obtained from the curves as illustrated in Fig. 31 and in Figs. 32 to 39, inclusive. Referring to Fig. 31, let z represent the value of $e\theta_y$ for which the corresponding values of DS_y and ES_y are required. The horizontal line through z cuts curve C at the point y, the abscissa value of which gives at once ES_y. The same horizontal line cuts the curve B at points s'

TABLE XXXV.

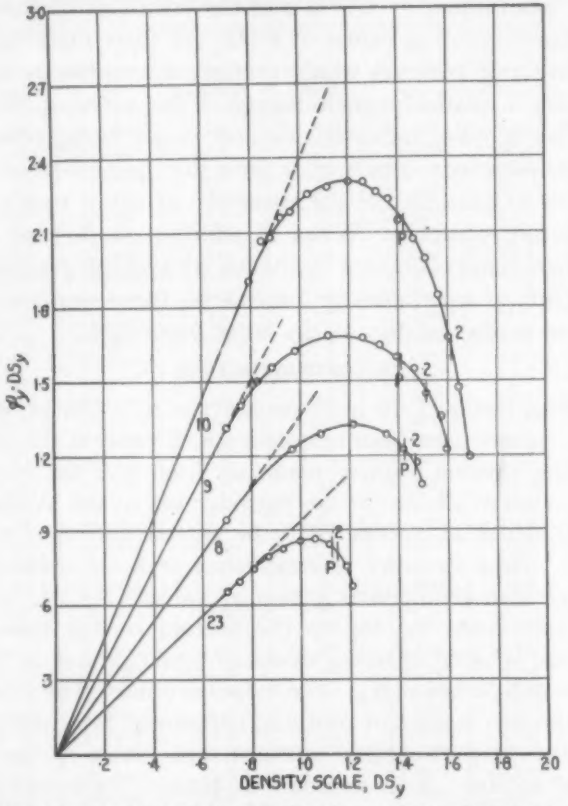
Data Used in Computing the Value of $\phi_y.DS_y$ for Various Portions of the Characteristic Curve Limited by Equal Gradients. These Values Were Used in Plotting the Curves in Fig. 50.

$e\theta_y$.	ES_y.	DS_y.	ϕ_y.	$\phi_y.DS_y$.
.0	2.10	1.60	.76	1.22
.1	1.85	1.58	.85	1.34
.2	1.61	1.52	.94	1.43
.3	1.40	1.47	1.05	1.54
.4	1.23	1.39	1.13	1.57
.5	1.08	1.33	1.23	1.63
.6	.95	1.26	1.33	1.67
.7	.85	1.18	1.39	1.65
.8	.76	1.12	1.47	1.64
1.0	.61	.99	1.62	1.60
1.2	.51	.89	1.74	1.55
1.4	.45	.81	1.80	1.42
1.6	.41	.76	1.85	1.41
1.8	.39	.73	1.87	1.36

and r' and perpendiculars through these points establish points s and r. The difference between the ordinates of points s and r is the required DS_y value. Using the curves for material No. 9, the values as shown in Table XXXV are determined and in the last column of this table the values of $\phi \cdot DS$ scale are given. By plotting these values as ordinates against the corresponding DS_y values as abscissa, curve No. 9 in Fig. 51 is obtained and proceeding in like manner curves for materials 23, 8, and 10 are plotted. It will be noted that $\phi \cdot DS_y$ increases at a constant rate until DS_y becomes equal to the density latitude of the material. Moreover this rate is exactly the same as that at which contrast increases (Fig. 50). The curves obtained then by using $\phi \cdot DS_y$

are identical to those of contrast up to the limit mentioned. From this point on, however, the relationship is very different. In the case of $\phi \cdot DS_y$ a maximum is obtained at a density scale value appreciably lower than D max. This is especially marked in the materials of higher contrast. Using such an evaluation it would

FIG. 51.



Curves showing the results obtained by evaluating the rate factor of contrast in terms of the mean slope computed on the basis of equal log exposure increments.

necessarily follow in the case of material No. 10 that the maximum contrast is obtained when a density scale of 1.2 is utilized. The D max. of this material is 1.67 and it is almost inconceivable that the maximum contrast is obtained when such a high proportion of the available density scale is discarded. The small vertical lines designated as p, drawn through the curves in Fig. 51, are

drawn at abscissa values equal to the density scale of the optimal positives made on the materials represented by the curves. The ordinate values at which these lines cut the curve therefore give the values of $\theta \cdot DS_y$ for the optimal positives. It will be noted that in all cases these are below the maximum value of $\theta \cdot DS_y$. The vertical lines designated as .2 represent the exposure scale of the materials limited by $G = .2$ and the ordinates of these points give the corresponding values of $\theta \cdot DS_y$ for these materials. Such an evaluation of contrast would produce an enormous change in contrast for a relatively small change in the value of the utilized DS_y. This is quite unreasonable and is not in agreement with practical experience. The author feels that this evidence of itself is sufficient to demonstrate the absurdity of using such a method for evaluating contrast. When in addition to this we consider the excellent results obtained by the use of average gradient evaluated in terms of equal density increments, there seems to be little question as to the validity of the latter procedure.

IX. CONTRAST SCALES.

As a final result of the work on contrast up to the present point numerical values have been obtained which express the total contrast of the various positive materials used and the contrast of these materials available for the reproduction of the Willow Pond test-object which is considered to be representative of a normal landscape. Now in order to establish a scale of contrast, these values have been arranged as shown in Table XXXVI, increasing progressively from the top to the bottom of the table. This arrangement is based upon the values of total contrast, as shown in the section designated as $\Omega_{.2}$. The values as obtained by multiplying together the two factors of contrast, extent and rate, without considering the proportionality constant, are given in the column designated as *Rel.* Now in order to convert these into absolute terms, it is only necessary to divide them by the square of the least perceptible difference, $l\triangle D$, which we shall assume is equal to .007, $.007^2 = .000049$ and we shall use for this value .00005, the reciprocal of which is 20,000. Hence, if we multiply the relative values shown in the first column by 20,000, the absolute values shown in the next column (*Abs.*) are obtained. It will be noted that the minimum contrast when expressed in absolute terms is 14,400 for material 1 and the maximum 73,800, for material 21. These num-

bers are clumsy for practical use and hence it is proposed that they be divided by 1000 in order to obtain values for practical use. These values are shown in the column so designated. These results bear simple relations both to the absolute and relative values since

TABLE XXXVI.

Values of Relative and Absolute Contrast for the Various Materials and Corresponding Values Expressed on a Scale Proposed for Practical Purposes.

Paper.	$\Omega_{.2}$			$\Omega_{.9}$			$\Omega_{.2}$
	Rel.	Abs.	Practical.	Rel.	Abs.	Practical.	14.4=1.00.
1	.72	14,400	14.4	.72	14,400	14.4	1
33	.83	16,600	16.6	.83	16,600	16.6	
2	.86	17,200	17.2	.84	16,800	16.8	
26	.87	17,400	17.4	.87	17,400	17.4	
37	.88	17,600	17.6	.87	17,400	17.4	
24	.89	17,800	17.8	.86	17,200	17.2	
23	.95	19,000	19.0	.93	18,600	18.6	
25	1.03	20,600	20.6	1.01	20,200	20.2	
20	1.09	21,800	21.8	1.09	21,800	21.8	
35	1.10	22,000	22.0	1.08	21,600	21.6	
30	1.15	23,000	23.0	1.13	22,600	22.6	
31	1.18	23,600	23.6	1.15	23,000	23.0	
14	1.27	25,400	25.4	1.25	25,000	25.0	
4	1.28	25,600	25.6	1.26	25,200	25.2	
32	1.32	26,400	26.4	1.30	26,000	26.0	
5	1.37	27,400	27.4	1.33	26,600	26.6	
15	1.44	28,800	28.8	1.39	27,800	27.8	
34	1.45	29,000	29.0	1.42	28,400	28.4	
22	1.49	29,800	29.8	1.47	29,400	29.4	
27	1.56	31,200	31.2	1.52	30,400	30.4	
8	1.58	31,600	31.6	1.55	31,000	31.0	2
36	1.64	32,800	32.8	1.60	32,000	32.0	
3	1.69	33,800	33.8	1.64	32,800	32.8	
28	1.82	36,400	36.4	1.76	35,200	35.2	
16	1.92	39,400	39.4	1.88	37,600	37.6	
9	2.03	40,600	40.6	1.96	39,200	39.2	3
17	2.08	41,600	41.6	2.02	40,400	40.4	
12	2.10	42,000	42.0	2.02	40,400	40.4	
6	2.20	44,000	44.0	2.15	43,000	43.0	
7	2.53	50,600	50.6	2.44	48,800	48.8	
18	2.63	52,600	52.6	2.59	51,800	51.8	
29	2.68	53,600	53.6	2.60	52,000	52.0	
10	2.79	55,800	55.8	2.69	53,800	53.8	4
19	2.98	59,600	59.6	2.89	57,800	57.8	
13	3.28	65,600	65.6	3.12	62,400	62.4	
11	3.46	69,200	69.2	3.33	66,600	66.6	5
21	3.69	73,800	73.8	3.60	72,000	72.0	

it is only necessary to multiply them by 1000 to obtain the absolute, or to divide by 20 in order to obtain the relative value in case this is desired. On the contrast scale proposed, therefore, the values for the group of materials used in this work extends from 14.4

to 73.8. This contrast scale thus provides for the expression of contrast of materials appreciably lower and higher without going below unity or exceeding 100. It is thought that such a scale is of maximum convenience for practical purposes. In choosing the positive materials used in this work an attempt was made to obtain those of lowest and highest contrast available, and it is doubtful whether materials ever will be available of much greater contrast, nor is it probable that materials of lower contrast will be of any great utility.

If now we wish to state the relative contrast of any two materials it is only necessary to determine the ratio of their contrast values. For instance, material 15 has a contrast value of 28.8, which is precisely twice that of material 1. It is interesting to note the relation between the contrast values of the group of papers 8, 9, 10, and 11. These are all glossy materials, the increase in contrast being due entirely to emulsion characteristics. The contrast of No. 8 is 31.6, which is approximately *twice* that of No. 1. That of No. 9 is 40.6, which is approximately *three* times that of No. 1. The contrast of material 10 is 55.8, this being about *four* times that of No. 1; while No. 11 has a value of 79.2, this being approximately *five* times that of No. 1. It will be noted therefore that the entire range of contrast in this group of materials is from one- to five-fold.

In the second part of Table XXXVI are given the contrast values for the portions of the characteristic curves utilized in making the optimal positives. It will be noted that these are in practically the same order as $\Omega_{.2}$ and that the difference between the values of $\Omega_{.2}$ and Ω_p increases progressively as the contrast increases, showing that as the contrast becomes greater, more and more of the available contrast scale is sacrificed in order to obtain optimal photographic quality. The relation of these contrast values to other characteristics of the materials and of the optimal positives, and the significance of these relations from the standpoint of tone reproduction will be discussed in the following section.

SUMMARY.

The results obtained in this section may be summarized as follows:

(*a*) Using the *statistical* data derived from section B relative to the characteristics of the *optimal* positives, and the *sensitometric*

data derived from characteristic curves for the *positive materials*, the portion of the characteristic curve utilized in making each optimal positive was determined.

(*b*) An inspection of the data relative to the utilized portions of the characteristic curve shows that the limiting gradient, θ_p, is not constant but depends upon the contrast of the positive material. Furthermore, the data show that there is no reason to believe that the limiting gradient in the high-light region is different in value from that in the shadow region for any particular optimal positive. It has also been demonstrated that the ratio of the shadow and high-light limiting gradients depends vitally upon the printing exposure used in making the positive. The increase or decrease in exposure required to make the limiting gradient equal at the extremes of the used portion is in all cases small and a reëxamination of the positives indicates that a change in the printing time which would equalize limiting gradient results in an improvement of photographic quality as judged from the standpoint of average print density.

(*c*) By applying the correction to printing exposure, the equalized limiting gradient, $e\theta_p$, for each of the optimal positives has been determined.

(*d*) The evaluation of the contrast function has been completed, giving the following formula for computing contrast in absolute terms:

$$\Omega = \frac{1}{(l\triangle D)^2} \cdot DS \cdot \overline{G}(D)$$

where $l\triangle D$ = least perceptible difference in density
= .007
DS = density scale
$\overline{G}(D)$ = average gradient evaluated with respect to equal density increments.

(*e*) A graphic method for evaluating contrast directly in terms of an enclosed area has been derived.

(*f*) By use of this graphic method numerical values of total contrast, $\Omega_{.2}$, have been determined for each of the positive materials and also values of partial contrast, Ω_p, for each of the optimal positives. In the former case the integration was carried out between the limits where $dD/d \log E = .2$, and in the latter between the points where $dD/d \log E = e\theta_p$.

(*g*) The relation between contrast and magnitude of density scale, DS_y, was shown graphically for typical cases.

(*h*) Experimental evidence has been presented showing that it is not possible to use directly the value of ϕ (gradient evaluated with respect to equal increments in log E) for the evaluation of contrast.

(*i*) A contrast scale for practical use has been proposed.

Brownian Movements of the Same Particle at Different Pressures and in Different Gases. S. KWARTIN. (*Zeit. f. Phys.*, **38**, No. 8.)—About two years ago Landman succeeded in keeping the same particle under observation at several pressures. In this paper for the first time the gas is changed in the space about a particle in Brownian movement without the permanent loss of it from sight. This was accomplished by the very careful flushing of the system with a second gas and by repeating this until the first gas had practically disappeared. There was a marked increase in the liveliness of the movements of the particle when it was surrounded by hydrogen in place of nitrogen or carbon dioxide. Similarly a reduction of pressure without any change in the nature of the gas caused a quickening of the motions of the particle. G. F. S.

Records of the Electric Field of the Atmosphere up to an Altitude of 20,000 Metres. P. IDRAC. (*Comptes Rendus*, June 28, 1926.)—Along with systematic studies of the conductivity of air in the high regions of the atmosphere a successful attempt has been made at the aerodynamic observatory located at Trappes to obtain records of the change of atmospheric potential with altitude. Six sounding balloons came down with records of this quantity. Of these three went higher than 13,000 metres, while one reached 20,000 metres. All ascensions were by day.

Wicks impregnated with lead nitrate were used to get the potential at a point. The balloon carried two such wicks separated by a vertical distance of several metres and insulated from the body and cordage of its carrier. Through the intermediary of a lamp with two grids, change of atmospheric potential was translated into the readings of a milliammeter photographically recorded.

In general the electric field in the atmosphere becomes less with increasing altitude. The author gives as the mean of his results 10.4 volts per metre at 4000 metres; 5.6 volts/metre at 6000 metres, and 2.3 volts/metre at 8000 metres. From the three balloons that entered the isothermal layer of the atmosphere, it was discovered that at the lower limits of this the field resumes a considerable value amounting to 30 or 40 volts/metre. The one balloon that reached 20,000 metres showed that above 16,000 there is a regular decrease, 12 volts/metre at 16,000 metres; 5 volts/metre at 17,000 metres and 1.2 volts/metre at 19,000 metres. G. F. S.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

SYPHONS AND MEASURING DEVICES FOR PHOTOGRAPHIC SOLUTIONS.[1]

By K. C. D. HICKMAN.

THE addition of photographic solutions to tanks in continuous processing machinery may be regulated by the footage of motion picture film.

To a master sprocket wheel a worm reduction gear is coupled and caused to actuate a stopcock which alternately opens and closes a pneumatic supply line to the outside air. The supply line may be filled with air under greater or lesser pressure than atmospheric provided by simple injector water pumps. The line is connected with the pump, the intermittent valve and a series of measuring vessels of special construction placed under or over the stock tanks of solution; the connections are all in parallel. Alternations of pressure cause the filling and discharge of the measuring units into the processing machinery. There are no valves or moving parts, the irreversible cycle being produced by liquid seals in bent tubes.

ANNULAR-OBLIQUE ILLUMINATION FOR THE MICROSCOPY OF SURFACE TEXTURE.[2]

By L. A. Jones.

THE visibility of surface texture and detail is vitally dependent upon the angle at which the illumination is incident. This paper describes a method of obtaining annular-oblique illumination of known and reproducible distribution. The annular source has appreciable extension in a direction perpendicular to the plane of the sample, thus giving graduated illumination of the surface cavities. Such illumination gives better modelling and less distor-

* Communicated by the Director.

[1] Communication No. 274 from the Research Laboratories of the Eastman Kodak Company and published in *Trans. Soc. Mot. Pict. Eng.*, No. 26, 1926, p. 37.

[2] Communication No. 285 from the Research Laboratories of the Eastman Kodak Company and published in *Paper Trade J.*, **83**, 56, 1926 (Oct. 14).

tion of detail than light from a source of small angular size incident from a single direction. Photomicrographs of various materials are shown illustrating the effect of varying the angular size of the source upon the rendition of surface detail.

COLOR SENSITIZING PHOTOGRAPHIC PLATES BY BATHING.[3]

By M. L. Dundon.

IN color sensitizing plates by bathing, it is necessary to get a certain required amount of sensitizing dye into intimate contact with the silver bromide grains of an emulsion without leaving an excess in the gelatin and especially without leaving any precipitated dye or scum on the surface. Some general hints on the technique of bathing plates and films are given together with specific directions for sensitizing with Acridine Orange, Erythrosine, Rhodamine B, Pinaflavol, Orthochrome T, Pinaverdol, Pinachrome, Pinachrome Violet, Pantochrome, Pinacyanole, Naphthacyanole, Dicyanine A, Kryptocyanine and Neocyanine. Accompanying spectrograms were all made on Eastman 40 plates sensitized according to the procedures recommended and are, therefore, comparable.

FLOCCULATION AND DEFLOCCULATION OF THE SILVER HALIDES.[4]

By S. E. Sheppard and R. H. Lambert.

THIS is a study of a particular phase of silver halide precipitation and emulsification, namely, the phases or aspects covered by the terms flocculation and deflocculation. Renwick has maintained that the process of silver bromide emulsion grain formation is essentially an electrical precipitation or coagulation regulated mainly by the concentration, both absolute and relative, of the two reacting solutions at each instant and each minute volume of the mixture. In this communication it is attempted to show that electric coagulation plays only a subordinate part in determining emulsion grain growth. Generally, in the absence of gelatin, flocculation phenomena occur which are, however, distinct in character from those of the typical insoluble colloids.

[3] Communication No. 287 from the Research Laboratories of the Eastman Kodak Company and published in *American Photography*, **20**, 670, 1926.

[4] Communication No. 279 from the Research Laboratories of the Eastman Kodak Company and published in *Colloid Symposium Monograph*, **4**: 281, 1926.

The secondary particles produced by this coagulation can be reversibly deflocculated and are not true primary bromide grains. In the presence of gelatin the threshold concentration of reacting salts for flocculation is progressively raised. It is shown experimentally that on washing silver bromide precipitate, formed in the presence of excessive silver bromide or silver nitrate, a partial deflocculation occurs, which is a maximum at a certain stage, and then decreases. The peptizing effect of potassium bromide solution is reversible, whereas that of the silver nitrate solution is irreversible. Potassium nitrate present after mixing has no peptizing and probably no coagulating effect upon the silver halide, but it prevents peptization by potassium bromide solution when added to thoroughly washed silver bromide before potassium bromide is added.

The experimental data obtained are discussed from the points of view of the coagulation theories of Lottermoser and Jablczynski. The latter has maintained that the whole process of flocculation and grain growth of silver halides is entirely a matter of recrystallization by diffusion of material dissolved from smaller grains and reprecipitated on larger ones. While these experiments support the view that the grain growth with silver halides is chiefly due to recrystallization, they show that in the absence of gelatin and similar protective colloids, the adsorption and coagulation postulated by Lottermoser are taking place in some degree. The existence of crystal aggregates, that is aggregates of partially fused together silver halide grains, is evidence that both processes are operative.

The relation of the two processes is discussed, and it is shown that, in the ripening of silver bromide in the presence of excess potassium bromide solution, the number of grains decreases in the first stages of the process according to the law similar for that found to hold for the rapid coagulation of colloids.

On the Diffraction of Light by Spherical Obstacles. C. V. RAMAN and K. S. KRISHNAN. (*Proc. Phys. Soc. London,* Aug. 15, 1926.)—If an opaque sphere intercepts a part of the light coming from a small source, there is in the centre of the resulting shadow a bright spot due to the diffraction of the light. It is usually assumed that the same result is obtained by replacing the sphere by a disc of the same diameter. The bright spot appears in both cases but there are marked differences, especially in its brightness.

A steel ball of quarter-inch diameter and an accurately made steel disc of the same diameter were used. These were mounted side by side about two metres from the source of light. When the eye was shielded from all light except that diffracted around the obstacles and was aided by a lens, the bright spot could be seen as close to the sphere as 3 cm. and at a distance of only 2 cm. from the disc. A series of photographs shows the spot to be much brighter behind the disc than behind the sphere. They show, further, that the general illumination is greater in the shadow of the disc than in that of the sphere. In addition, the bright spot behind the sphere appeared reddish in comparison with that behind the disc.

An Abney rotating sector photometer was used to compare the relative intensities of the two bright spots, the source of light being 232 cm. in front of the opaque bodies. While the results are not very accurate owing to the difference in color, they suffice to bring out the great difference in the intensities. At a distance of 100 cm. behind the obstacles the bright spot behind the disc has about one and a quarter times the intensity of that behind the sphere. As the point of observation approaches the objects the ratio grows slowly larger until at 20 cm. it becomes 3. Upon further approach it rises rapidly in value, reaching 31.6 at a distance of 6 cm.

A qualitative explanation of this difference may be found in the following. Let the centres of the sphere and of the disc be placed at the same distance from the source of light. In the case of the disc, rays come to the edge from which diffraction takes place directly. It is not so with the sphere. A cone drawn from the source tangent to the sphere does not meet the latter along its equator but along some other circle nearer to the source. Before the light incident along this circle can get to the other side of the sphere and send out waves to produce the central bright spot, it must, in some way, creep over a part of the circumference. This distance is greater the nearer the spot is to the sphere. "Further, the luminous edge of the sphere is found to diminish in brightness much more rapidly than in the case of the disc when the observer's eye is moved laterally into the region of shadow." G. F. S.

Use of Potassium Bi-iodate in Volumetric Analysis. I. M. KOLTHOFF and L. H. VAN BERK, of the University of Utrecht (*Jour. Am. Chem. Soc.*, 1926, **48**, 2799–2801), recommend the use of potassium bi-iodate for the standardization of standard solutions of thiosulphate and of alkalies. It is readily obtained in a pure state by heating together a solution of potassium chlorate, concentrated hydrochloric acid, and powdered iodine, filtering and cooling the filtrate. Crystals of potassium bi-iodate deposit, and are purified by recrystallization from water two or three times. This salt has a high equivalent weight, and its iodic acid content behaves as a very strong acid.

J. S. H.

NOTES FROM THE U. S. BUREAU OF MINES.*

LIMITS OF INFLAMMABILITY OF GASES AND VAPORS.

By H. F. Coward and G. W. Jones.

THE Bureau of Mines, in coöperation with the Safety in Mines Research Board of Great Britain, is studying physical and chemical factors connected with the initiation and propagation of flame in different gases under various conditions. The limits of inflammability of mine gases, particularly methane and carbon monoxide, are of fundamental importance in the study of mine explosions and their prevention. The inflammable limits of various gases and vapors are of great importance in metallurgical and other industries, but the data are scattered and often contradictory. The authors have made a critical review of all figures published on the limits of inflammability of combustible gases and vapors when in admixture with air, oxygen, or other atmospheres. Suspended dusts and liquid mists were not considered. The chosen values for certain gases in air, at ordinary temperatures and pressures, are as follows: Methane, 5.3 to 14 per cent., and in turbulent mixtures, 5.0 to 15 per cent.; ethane, 3.2 to 12.5; propane, 2.4 to 9.5; butane, 1.9 to 8.5; hydrogen, 4.1 to 74; carbon monoxide, 12.5 to 74 per cent. Approximate values were determined for many other gases.

TEMPERATURES IN FURNACES BURNING POWDERED COAL.

By Ralph A. Sherman.

THE Bureau of Mines, in coöperation with the American Society of Mechanical Engineers, is investigating the proper application of boiler-furnace refractories. As part of this investigation, fundamental data have been obtained on temperatures in furnaces equipped with extended radiant heat-absorbing surface and different types of powdered coal burners. Conditions of service for refractory linings burning powdered coal are generally regarded as more severe than in furnaces fired with stokers.

The two principal methods of reducing the severity of the refractories service are direct cooling by admitting through chan-

* Contributed by the Director.

nels back of the lining, part of the air required for combustion, and indirect cooling by supplementing the heat-absorbing surface of the boiler tubes with sidewall heating surface, water-tube slag screens, and radiant super-heaters. The temperature in the furnace depends, among other causes, partly on the area of exposed surfaces that absorb radiant heat, so that increasing the heat-absorbing area tends to lower the mean temperature of the furnace gases. In the bureau's experiments comparative data were obtained on the temperature distribution with the different types of fuel burners tested, and on the temperatures attained in these furnaces.

Further details are given in a report to be published by the *Journal* of the Society.

FELDSPARS OF THE PACIFIC NORTHWEST.

By Hewitt Wilson.

THE Bureau of Mines has investigated the possible sources of feldspar and flint in the Pacific Northwest that may be of commercial use in the manufacture of ceramic products. The data were obtained in coöperation with the University of Washington, which is conducting a study of the ceramic properties of kaolins of Washington and Idaho at the present time. It is important, in respect to utilization of the kaolins, that sources of feldspar and flint be considered, in order to have the three principal components of white ware. It was found that the flint could be most easily produced by grinding the purified quartz sand washed from the kaolins. The feldspars discovered in the Northwest are soda feldspars, with the exception of two small deposits of potash feldspar. Deposits of commercial grade are found on Orcas Island, Washington, near Avon, Idaho, on Ashland and Gold Creek, Oregon, and Big Lake, Washington. Deposits are also known to exist near Yale, British Columbia, and on Douglas Channel, British Columbia. The data obtained indicate that the Pacific Northwest will be largely dependent on soda feldspar. The deposit at Deer Harbor is with respect to quantity, ease of mining and transportation, the most important source of feldspar for a future white-ware industry.

The results will be published in more detail by the Bureau of Mines.

BENTONITE AND WELL DRILLING.

By C. W. Davis.

In drilling most wells while searching for oil, potash, or other products, water is used. If bentonite (a clay-like substance) be encountered, drilling difficulties arise when water comes in contact with bentonite. Although it had been demonstrated that if a saturated solution of common salt replace the water such difficulties for a certain bentonite would be removed, the action of other liquids on different bentonites had not been studied. The results of a considerable number of experiments indicate that motor oil, kerosene or gasoline used instead of water would prove ideal in the prevention of difficulties experienced in drilling all varieties of bentonite if water could be excluded (the latter in the presence of bentonite would form oil-water emulsions which would be difficult to handle) and that where water must be used the addition of some salt will prove a satisfactory remedy, the quantity and nature of the salt being specific for each variety of bentonite.

On the Relation between Barometric Pressure and Gas Pressure in Mines. Henry Harries. (*Monthly Notices, Roy. Astron. Soc.*, Geophys. Sup., June, 1926.)—For generations miners in Great Britain and on the continent believed that gas came into mines during quiet weather before disturbed conditions showed themselves at the surface of the earth. Then early in the nineteenth century, seemingly without the support of experimental evidence, the former view yielded to the opinion that explosions occurred under a low and falling barometer. This appeared natural enough, since a decrease of external pressure should permit the explosive gas to expand and occupy space previously free from it. However, after the Mines Act of 1872 had required the keeping of a barometer at the mouth of gaseous mines, it was observed that gas escaped into the workings while the barometer was rising. Thus on both sides of the Atlantic the barometer was rejected as valueless in predicting a flow of gas.

It seems that it is almost impossible to get records of gas pressure from individual collieries, yet two records have been obtained for the same period and for bodies of gas sealed off in mines 150 miles apart and having a difference of 900 feet in distance below sea-level. A comparison of the water gauges registering the pressure of the two gas volumes in relation to that of the atmosphere shows that they pursue remarkably parallel courses. The great intervening distance precludes the possibility of local conditions affecting the result. The

data in relation to the Seaham Colliery, published by Corbett in 1883, is discussed and perfect correlation is claimed between "(1) the behavior of distantly situated volumes of gas in the earth's crust; (2) a rising barometer and increasing pressure of gas in a sealed-up chamber; (3) the gas pressure in a sealed-up chamber and the escape of gas in an open chamber; and (4) a rising barometer and the escape of gas." The author regards the slight downward yielding of the earth's surface under high barometric pressure and the rising of the surface when the pressure grows less as responsible for changes in the flow of gas within the earth. "As the result of the compression of the strata under high-pressure conditions, the earth's surface assumes a slightly concave formation, so that the vents or stomata of the outer skin close, and the cracks, cleats, cleavages, etc., in the strata widen, of course very slightly, but sufficiently to enable the gas streams to follow a general downward course along all available channels, invading open mine workings as well as over-charging closed caverns and chambers. As the high pressure passes away, the weight of the superincumbent atmosphere diminishes, the barometer falls, and the earth's surface swells and becomes slightly convex above the normal level; the vents reopen, and the gas streams reversing their direction mount upward, drawn toward the region of least pressure. During this low-pressure stage there have been notable instances of the complete cessation of gas escaping into the mine galleries." In support of the motion of the crust of the earth he cites evidence of the actual visible distortion of roofs of rock and the discovery of pockets of gas under considerable pressure.

G. F. S.

Semi-Centennial of the Connecticut Agricultural Experiment Station.—The Connecticut Agricultural Experiment Station was founded by an appropriation by the General Assembly of that State in 1875, and was incorporated by an act of March 21, 1877, as a separate and independent state institution to be located at New Haven, and "to promote agriculture by scientific investigation and experiment." It was the first agricultural experiment station established in the United States. Unlike other stations, it is not connected with any other institution. The first director was Wilbur O. Atwater, noted for his work on the composition of American foodstuffs and on animal calorimetry. The second director was Samuel W. Johnson, a pioneer in the study of fertilizers and of the nutrition of plants. The fiftieth anniversary of the founding of the station was celebrated by exercises held at New Haven on October 12, 1925. The station has published a monograph (Connecticut Agricultural Experiment Station Bulletin 280, 1926, 615–657) which contains an historical sketch of the station, certain of the addresses delivered at the semi-centennial celebration, and portraits of its directors.

J. S. H.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting Held Wednesday, December 15, 1926.)

THE stated monthly meeting of the Institute was called to order by the President, Dr. Wm. C. L. Eglin, at eight-fifteen P.M.

The Secretary announced that the minutes of the November meeting had been printed in full in the December number of the JOURNAL of the Institute and therefore moved that they be approved as printed. The motion was seconded and unanimously adopted.

The Secretary announced that the following additions to the membership of the Institute had been made: One life member; seventeen resident members; eight non-resident members; four student members; a total of thirty.

He announced with regret the death of five members, and stated that proper citation of these deaths had been made in the JOURNAL.

The Secretary also made a statement concerning the Christmas Week Lectures to be given by Prof. R. W. Wood, on "Recreations with Radiations," and stated that a prize of fifty dollars was offered by the Institute to that student who offered the best book of lecture notes taken upon the series of four lectures, the book to be presented at the close of the last lecture. He suggested the suitability of a ticket to these lectures as a Christmas present for any young person.

This being the meeting at which nominations of officers and members of the Board of Managers for the ensuing year must be made, the Secretary presented the following nominations, which had been offered by various members:

Office.	Name.	Term.	Nominated by:
President	Dr. Wm. C. L. Eglin	One year	Dr. Walton Clark Mr. M. S. Morgan
Vice-president	Mr. Walton Forstall	Three years	Mr. Nathan Hayward Mr. Henry Howson
Treasurer	Mr. Benjamin Franklin	One year	Mr. Walton Forstall Mr. Nathan Hayward
Board of Managers:	Mr. Charles E. Bonine Mr. Edward G. Budd Dr. Walton Clark Mr. Charles Day Mr. Clarence A. Hall Dr. G. A. Hoadley Mr. James S. Rogers Mr. Haseltine Smith	Three years	Mr. M. S. Morgan Mr. Benjamin Franklin
	Mr. Samuel T. Wagner (for the unexpired term of Mr. F. J. Chesterman)	One year	Mr. M. S. Morgan Mr. Benjamin Franklin

The President called for any additional nominations from the floor. No such nominations were offered and upon motion adopted, the nominations were declared closed.

There being no further business, the President presented as the speaker of the evening, Mr. Nevin E. Funk, Operating Engineer of the Philadelphia Electric Company, who spoke upon the subject, "Unusual Engineering Features of the Conowingo Dam and Power Plant."

Mr. Funk presented, to a large and attentive audience, a most interesting paper, which showed admirably the planning and development of this splendid engineering project. His paper was followed by numerous questions from the floor and some discussion.

The meeting adjourned with a rising vote of thanks in appreciation of Mr. Funk's paper, at nine-forty-two P.M.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting Held Wednesday, December 1, 1926.)

HALL OF THE COMMITTEE,
PHILADELPHIA, December 1, 1926.

MR. CLARENCE A. HALL, *in the Chair.*

The following reports were presented for first reading:

No. 2857: Prof. G. E. Beggs' Method of Determining Stresses in Indeterminate Structures.

No. 2860: An Electrical Power Drive for Typewriters.

GEORGE A. HOADLEY,
Secretary to Committee.

BARTOL RESEARCH FOUNDATION.

ON THE ABSORPTION OF β-RAYS BY MATTER.

TO THE EDITORS:

It has been shown by Georges Fournier[1] that the mass coefficient μ/ρ of β-rays by the chemical elements follows a very simple law, namely:

$$\mu/\rho = a + bN, \qquad (1)$$

where N is the atomic number of the element, a and b are constants for any one kind of rays but are different for different rays; further, the ratio a/b is approximately constant and equal to 105 for β-rays of very different penetrating power.

[1] G. FOURNIER, *C. R.*, **180**, pp. 284 and 1490, 1925. Also, (Mrs.) J. S. LATTES and G. FOURNIER, *C. R.*, **181**, pp. 855 and 1135, 1925.

Further, he has shown[2] that μ/ρ for compounds is given by the following additive law:

$$\frac{\mu}{\rho} = \frac{xP_x(\mu/\rho)_x + yP_y(\mu/\rho)_y}{M}, \tag{2}$$

where $(\mu/\rho)_x$ and $(\mu/\rho)_y$ are the mass coefficients of absorption by the elements X and Y.

P_x and P_y are the atomic weights of the elements X and Y.
x and y are the number of atoms of the elements X and Y.
M is the molecular weight of the compound and is equal to $xP_x + yP_y$.

We can generalize this expression, which becomes, when the values of $(\mu/\rho)_x$, $(\mu/\rho)_y$, $(\mu/\rho)_z$, . . . given by (1) are inserted:

$$\frac{\mu}{\rho} = a + b\frac{xP_xN_x + yP_yN_y + zP_zN_z + \cdots\cdots}{xP_x + yP_y + zP_z + \cdots\cdots}$$

$$= a + b\frac{\Sigma xP_xN_x}{\Sigma xP_x}. \tag{3}$$

where N_x, N_y, N_z, . . . are the atomic numbers of the elements X, Y, Z, . . .

Now according to (2) or its more general expression (3), the values of the mass coefficient of absorption for isotopic compounds (that is, compounds containing the same elements but different isotopes of one or more of the elements), such as $HCl_{(35)}$ and $HCl_{(37)}$, should be different. This seems to me to be very improbable in view of the fact that the mass coefficient of absorption of β-rays by elements is proportional to the atomic number and not to the atomic weight as discovered by Fournier. Hence, I believe that, in a molecule, each atom contributes its coefficient of absorption proportionally to its atomic number and not proportionally to its atomic weight as Fournier assumes. Then, in replacing in (3), P_x, P_y, P_z, . . . by N_x, N_y, N_z, . . . we obtain:

$$\frac{\mu}{\rho} = a + b\frac{\Sigma xN_x^2}{\Sigma xN_x}. \tag{4}$$

We can regard $\Sigma xN_x^2/\Sigma xN_x$ as the equivalent atomic number of the molecule for the absorption of β-rays. In the case where the atomic numbers of the constituent atoms of a molecule are approximately equal, we can simplify the formula (4), namely:

$$\frac{\mu}{\rho} = a + b\frac{\Sigma xN_x}{\Sigma x}. \tag{5}$$

The following table gives the values of the coefficient of absorption of the β-rays of RaE calculated according to (3), (4), and (5), as well as the experimental values found by Fournier, who found the constants a and b to be 15.0 and 0.142, respectively, these values being used in the calculations:

[2] G. FOURNIER, *C. R.*, **183**, pp. 37 and 200, 1926.

Substance.	μ/ρ Calculated According to			μ/ρ
	(3)	(4)	(5)	Experimental.
SiO_2	16.5	16.5	16.4	16.3
MnO_2	17.7	17.6	16.9	17.8
MgO	16.5	16.5	16.4	16.5
Al_2O_3	16.5	16.5	16.4	16.8
HBO_3*	16.0	16.1	15.9	16.2
KBr	19.2	19.2	18.8	19.2
$NaNO_3$	16.2	16.2	16.2	16.2
KNO_3	16.7	16.7	16.4	16.7 (?)
$C_{10}H_8$	15.8	15.8	15.5	15.8
$C_6H_2(OH)(NO_2)_3$	16.0	16.0	15.9	15.9
CCl_3	17.3	17.3	17.1	17.1

* This is the formula given by Fournier, *C.R.*, **183**, p. 502, 1926, but it is very probable that the substance used by Fournier was boric oxide, B_2O_3, the calculated values of μ/ρ being all 16.0 for B_2O_3.

We see that the results given by the formula (5) are just as accurate as, if not more accurate than, those given by the less logical formula (2) or its equivalent (3). Also, in cases where the atomic numbers of the constituent atoms of a molecule do not vary greatly, the results calculated according to the formula (4) agree very well with those of experiment, otherwise they are too low.

However, the substances called abnormal by Fournier show experimental values of μ/ρ much greater than those calculated by any formula yet proposed.

HERBERT J. BRENNEN, D.Sc.,
Bartol Research Fellow.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, December 8, 1926.)

RESIDENT.

MR. C. REED CARY, Vice-president, Leeds and Northrup, 4901 Stenton Avenue. For mailing: 1 Lehman Lane, Germantown, Philadelphia, Penna.

MR. DANIEL O. LANDIS, Experimenter in Electrical Instruments, 261 South Fourth Street. For mailing: 1822 North Bouvier Street, Philadelphia, Penna.

MR. OTTO T. MALLERY, Economist, 112 South Sixteenth Street, Philadelphia, Penna.

MR. WILLIAM PLASS, Salesman, 821 Arch Street. For mailing: 2103 North Eighteenth Street, Philadelphia, Penna.

MR. HAROLD RANKEN, Chief Engineer, Leeds and Northrup Company, 4901 Stenton Avenue. For mailing: 5046 Ehringer Place, Germantown, Philadelphia, Penna.

MR. CHARLES S. REDDING, Treasurer, Leeds and Northrup Company, 4901 Stenton Avenue. For mailing: 240 Wyncote Road, Jenkintown, Penna.

NON-RESIDENT.

DR. E. P. ADAMS, Professor of Physics, Princeton University. For mailing: 12 Nassau Street, Princeton, N. J.

MR. DONALD GUNN, Engineer, 435 Sixth Avenue, Pittsburgh, Penna.

MR. HOWARD W. MATHESON, Chemist and Chemical Engineer, 611 Power Building, Montreal, Quebec, Canada.

STUDENT.

MR. KENNETH GIBSON, Machinist, Cramp Shipyard. For mailing: 2843 North Eleventh Street, Philadelphia, Penna.

MR. WILLIAM HURST, Apprentice, Bartol Laboratories. For mailing: 2513 North Cleveland Avenue, Philadelphia, Penna.

MR. JULIUS KRETZ, JR., Student. For mailing: 714 State Street, Camden, N. J.

MR. WILLIAM A. OLSEN, Assistant, Physical Research Laboratory, Victor Talking Machine Company, Camden, N. J. For mailing: 812 North Eighth Street, Camden, N. J.

CHANGES OF ADDRESS.

MR. EDWARD G. BUDD, 157 Pelham Road, Germantown, Philadelphia, Penna.

MR. W. A. DOUGLASS, 28½ Ziegler Tract, Penns Grove, N. J.

MR. RALPH L. GOETZENBERGER, 8016 Flourtown Road, Chestnut Hill, Philadelphia, Penna.

MAJOR THOMAS H. GRIEST, 623 Westview Avenue, Mt. Airy, Philadelphia, Penna.

MR. EDWARD F. HENSON, Lafayette Building, Fifth and Chestnut Streets, Philadelphia, Penna.

MR. N. G. HERRESHOFF, P. O. Box 116, Coconut Grove, Fla.

MR. CHARLES H. HERTY, In care of The Chemical Foundation, Inc., 85 Beaver Street, New York City, N. Y.

PROF. CHARLES E. LUCKE, Mechanical Engineering Department, Columbia University, New York City, N. Y.

DR. ALFRED OBERLE, 500 Keith and Perry Building, Kansas City, Mo.

MR. FREDERICK W. SALMON, P. O. Box 1104, Birmingham, Ala.

MR. ARTHUR W. THOMPSON, President, The United Gas Improvement Company, N. W. corner Broad and Arch Streets, Philadelphia, Penna.

MR. E. B. TUTTLE, 1631 Arch Street, Philadelphia, Penna.

MR. WALTER C. WAGNER, 2301 Market Street, Philadelphia, Penna.

NECROLOGY.

Mr. Carl Ethan Akeley, New York City, N. Y.

LIBRARY NOTES.

RECENT ADDITIONS.

American Annual of Photography, 1927. Volume 41. 1926.

Annalen der Physik. Register zu Band 31–78 (1910–1925) der vierten Folge. 1926.

BARNARD, WM. N., ELLENWOOD, F. O., and HIRSHFELD, C. F.—Elements of Heat-Power Engineering. Part I. Thermodynamics and Prime Movers. Third edition of first part of Hirshfeld and Barnard's Elements of Heat-Power Engineering, completely rewritten by F. O. Ellenwood and W. N. Barnard. 1926.

Beilstein's Handbuch der organischen Chemie. Vierte Auflage, Neunter Band. 1926.

BELLUZZO, GIUSEPPE.—Steam Turbines. Translated by A. G. Bremner. 1926.

BENEDICKS, CARL.—Metallographic Researches: Based on a Course of Lectures Delivered in the United States in 1925. First edition. 1926.

Forschungsinstitut der Czechoslovakischen Zuckerindustrie in Prag. Berichte für das Jahr 1925–1926. Volume 29. 1926.

FRIEND, J. NEWTON, EDITOR.—Text-book of Inorganic Chemistry. Volume 7, Part 3. 1926.

GARDNER, WILLIAM.—Chemical Synonyms and Trade Names. Third edition, revised and much enlarged. 1926.

HULL, H. B.—Household Refrigeration. Second edition enlarged. 1926.

HUMPHREYS, W. J.—Rain Making and Other Weather Vagaries. 1926.

MCCULLOUGH, ERNEST.—Practical Structural Design in Timber, Steel and Concrete. Third edition, revised and enlarged. 1926.

MOTZ, WILLIAM H.—Principles of Refrigeration. 1926.

PACORET, E.—Aide-Mémoire-Formulaire de la T.S.F. Edition 1926.

SCHMIDT, JULIUS.—A Text-book of Organic Chemistry. English edition by H. Gordon Rule. 1926.

SHERMAN, HENRY C.—Chemistry of Food and Nutrition. Third edition rewritten and enlarged. 1926.

SPELLER, FRANK N.—Corrosion: Causes and Prevention. First edition. 1926.

Zeitschrift für anorganische und allgemeine Chemie. Generalregister der Bände 101–150 (1917–1925). 1926.

OBSERVANCE OF FRANKLIN'S BIRTHDAY.

THE attention of the members of the Institute is called to the celebration of the two hundred and twentieth anniversary of the birth of Benjamin Franklin which will be held on January 17, 1927. Luncheon will be served at The Poor Richard Club, followed by a parade to Franklin's grave, participated in by representatives and members of the various civic organizations with which Franklin's name is associated, naval forces, and city organizations. Speakers of standing will address the gathering at the luncheon and at the grave.

All members of The Franklin Institute are invited to participate in a body. Those interested are requested to communicate with the Secretary.

HOWARD MCCLENAHAN,
Secretary.

BOOK REVIEWS.

CLINICAL ASPECTS OF SUNLIGHT AND ARTIFICIAL RADIATION. By Edgar Mayer, M.D. xvi–468 pages, numerous illustrations in the text and 36 plates, 8vo. Baltimore, The Williams and Wilkins Company, 1926. Price, $10.

This is a very extensive and careful review of the data that have been accumulated concerning the general hygienic effects of light and the therapeutic applications thereof. That light is the basis of life may be conceded, even though there are numerous instances of growth in the dark, just as oxygen may be considered the fundamental element of living processes, although anaërobic organisms are known. Full judgment on the data presented in the book cannot be made until a much larger experience has been accumulated and more observers have been able to review the methods. That light must have a beneficial effect seems to require no argument. Its direct relation as noted above to the great majority of living organisms establishes that fact. We are not astonished to learn that the value of sunlight was appreciated in very early times. It seems indeed that most, if not all, of religious cults go back to sun worship. The pious man who faces the east in his prayer, as is the case in so many forms of worship, perpetuates the ancient custom. Our principal holidays—those that are not commemorative of some great leader or national incident—are seasonal in their origin, governed essentially by the sun's course. It is curious to note that since all such feasts and fasts originated in the northern hemisphere, the colonization of the southern hemisphere by northern races has brought about a dislocation of dates, sometimes in an almost ludicrous way. In New Zealand they wear overcoats on the Fourth of July and go picnicing on Christmas. The great Jewish feast of the harvest home falls there in the spring.

Sunlight, however, is not now the sole reliance for physiologic and therapeutic effects. It is now known that rays similar to those we call light but invisible to us abound in the light of space, but a small proportion of such rays reaching us. Research has given us means of going far beyond the natural light-producing waves—as they are designated—too small and too large for us to appreciate by our eyes.

In this work the principal attention is given to the nature of the waves beyond the violet and their effects. Comparatively little space is given to the infra-red. This may be due to the lack of effect as yet observed by such rays. Claims have been made for them, however.

Some description is given of the forms of apparatus used for producing the invisible rays, but it seems that there is still a lack of comprehensive and detailed information on this subject, except such as is given in lists of the makers. Some years ago a form of apparatus claiming to give a considerable supply of infra-red rays was advertised in one of the leading medical journals of the United States, but the writer of this review could not obtain any satisfactory evidence of action in this respect.

Many years ago a Philadelphian experimented with a green-house glazed with blue glass, and claimed that plants grew better under such conditions than with colorless glass. He presented his results in a book, but it aroused

only amusement. Yet there may be something in the filtration of the light. It may be that the long waves of the spectrum have an antagonistic action on the short waves. If a surface coated with luminous paint (calcium sulphide) is activated strongly and then exposed again to the activating light with a piece of red glass over part of the surface, it will be found that the luminosity is much diminished under the glass.

The work, as usual with present-day books, bases explanations on the undulatory theory of light. This seems to be going by the board, and one wonders what methods of classification and explanation will be available if anything like the corpuscular theory gains favor. The data in the book are almost entirely interesting to physicians. To them it is a very valuable compilation and should find wide use among those engaged in treating general diseases, especially diseases dependent on defective nutrition or lack of opportunity for free exposure to sunlight.

An interesting personal note is struck when the publisher attaches in a fly-leaf the names of those who have been concerned in the making of the book. One of the encouraging features of the present industrial age is the tendency to take the "people into partnership" to use the words that a Greek politician used many centuries ago, in defining the characteristics of democracy as exemplified in Athens.

Credit is given to Scheele for the discovery that light is the cause of the darkening of silver chloride, his observations being dated in 1777, but fifty years before Heinrich Schulze demonstrated by a very simple experiment that the darkening of silver chloride, which before his time had been attributed to air and heat, was really due to light. HENRY LEFFMANN.

AIDE-MÉMOIRE-FORMULAIRE DE LA T. S. F., THÉORIQUE ET PRATIQUE. RADIOTÉLÉGRAPHIE-RADIOTÉLÉPHONIE-RADIOTÉLÉMÉCANIQUE-RADIOTÉLÉVISION, Edition 1926. xiii, 577 pages, cloth, 17 × 13 cm. Paris, Librairie Scientifique, Albert Blanchard. Price, 32 francs.

A few years prior to the World War, wireless telegraphy had reached an important state of development in its application to communication of ships at sea with each other or with land stations. Reports of successful receptions made a strong appeal to the public mind, particularly those which resulted in averting marine disasters. Amateurs everywhere applied their energies to experimenting with this new kind of telegraphy and wireless of the damped wave variety in the Morse code became a widespread hobby. In the hands of men of science the subject progressed rapidly, undamped waves and thermionic tubes, and finally broadcasting were developed into the radio of to-day. With all the activities that have brought the art to its present satisfactory state, an immense amount of data on the electrical properties of the elements of transmitting and receiving apparatus have been accumulated and from these an applicable code of practice has gradually been evolved.

In the thirteen chapters of this reference manual, well-nigh everything in radio that we find in current use is discussed. The opening chapters contain a collection of formulas relating to the component elements of radio-apparatus, which is followed by a condensed account of the principles of electricity and magnetism. Next follows a chapter on the nature and propagation of electric

waves, the production of electric oscillations by means of tubes, and transmitting and receiving circuits which include a number of recent circuits and auxiliaries. Not much stress, however, is laid upon apparatus, diagrams mainly are depicted with fundamental data. Many other topics including radio-measurements are discussed at considerable length, as well as such matters as telegraphy by means of ultra-violet light and radio-telegraphy of pictures.

Although there is a satisfactory table of contents, a full index would add greatly to the value of a book of this kind which will be used for reference rather than as a text. Actual use is almost the only test by which its value can be properly determined. That is not difficult to carry out and at little outlay.

LUCIEN E. PICOLET.

PRACTICAL STRUCTURAL DESIGN IN TIMBER, STEEL AND CONCRETE. A text and reference work for engineers, architects, builders, draftsmen and technical schools; especially adapted to the needs of self-tutored men. By Ernest McCullough, C.E., Ph.D., Consulting Engineer. Third edition, revised and enlarged. 416 pages, 21.6 × 14.6 cm., illustrated, cloth. New York, Scientific Book Corporation, 1926. Price, $4 net.

The progress of applied science has perhaps been the most important stimulus to the flood tide of educational activity which has covered the country in recent years. In many instances, a growing widespread interest in cultural achievement has doubtless been the incentive, but in view of the handsome material reward that has often come as a result of thorough intellectual training, especially in the field of science and technology, the prospect of ultimate profit is the most likely lure to that most wholesome of speculations.

With the advancement of educational activity, text-books on every branch of technology have become available to those who, lacking the opportunity of college instruction, would perfect their knowledge of their vocations and so improve their condition. Unfortunately for students of that type certain prerequisites are essential for the profitable perusal of college texts, and either they must in one way or another satisfy that requirement or they must be provided with texts in which the derivation of applicable results is presented in summarized form. Over a considerable period of years many texts of that sort have appeared, particularly on structural design and kindred subjects. Doctor McCullough's book is one of these which differs from most of the others by the inclusion of an unusually ample collection of practical technique and details of procedure, thus affording a convenient and condensed source of information for the practitioner as well as providing a text for instruction.

The author states his belief that 95 per cent. of the work done in the design of structures can be explained to men whose knowledge of mathematics does not extend beyond that taught in high school. The subject is developed in accordance with that specification and necessarily many of the results of analytical derivation are given without proof. The ground covered is very extensive, every usual form of structure and material is discussed. A generous number of cases of beams with a variety of loadings is considered, with special conditions for metal, wood and reinforced concrete dwelt upon at length. Methods of evaluating the stresses in common types of bridge and roof trusses receive ample consideration along with many details of joints and connections,

especially those of wooden trusses. A chapter on graphic statics elucidates that method with a consideration of truss types in adequate diversity. Columns, structures, footings, tanks, chimneys and retaining walls constitute a well-balanced chapter of formulated design data and general information. The author's active contact with current practice makes the latter of especial value. The account of these numerous details which are impossible to formulate is characteristic not alone of this chapter, but occurs in varying degrees in every section of the work. The final chapters, semi-rigid frames, miscellaneous data and notes on reinforced concrete contain an equally valuable fund of qualitative data and method.

Not every teacher of students who are limited to high school mathematics will endorse the author's mode of presentation of the theoretical parts of the subject, but will adhere closely to the formal methods of the college type of text-book. For use of designing engineers, this departure from conventional methods is of no important consequence, and they will find an abundance of information on many by-ways of the subject of great practical value and interest.

LUCIEN E. PICOLET.

A TEXT-BOOK OF ORGANIC CHEMISTRY. By Dr. Julius Schmidt, Technical High School, Stuttgart; English Edition, H. Gordon Rule, Ph.D., D.Sc., University of Edinburgh. xxiv–798 pages, large 8vo. New York, D. Van Nostrand Company, 1926. Price, $9.

This work covers the field of organic chemistry in about the manner and to the same extent as the general run of modern manuals. Compounds are described in the usual order terminating with the complex group of proteins and their allies as found in living organisms. It might be well for authors of such manuals to agree to eliminate the last phase of organic chemistry. The chapters usually have a sort of "tired feeling," since the majority of workers in the modern organic research laboratory do not deal much with the subject. "Bio-chemistry" has now reached a high development and many workers are engaged in it, especially in connection with physiologic and pathologic laboratories. Notice, however, must be given of one feature of the section on the biologic compounds, namely, the comprehensive account of the chemistry of chlorophyl. This phase of bio-chemistry is generally neglected in ordinary manuals, because the animal proteins are of greater importance, in the usual applications of chemistry, medicine, pharmacy and hygiene. Chlorophyl plays a very important part in the phenomena of life. It may be said, indeed, that it is the fundamental "life-giver," for plants are primary foods. It has been long supposed that iron is an essential constituent of the chlorophyl molecule, but present knowledge, due largely to Willstätter's researches during the past fifteen years, is that magnesium is the metallic adjunct; phosphorus and iron are absent. The characteristic coloring matter of plants thus contrasts strongly in composition with that of animals. The absence of iron in chlorophyl makes more difficult the explanation of the curious disease of grazing cattle in New Zealand which has been so extensively studied by B. C. Aston. Cattle pastured on soils derived largely from volcanic dust suffer a sort of starvation, although the grass seems healthy and nutritious. Cattle will "die amid plenty" in such regions. Removed early to pastures on other soils they

may be saved, but Aston found that by furnishing a "lick" of molasses containing ferric ammonium citrate, the trouble could be averted. The matter was taken up at the meeting of the British Association for the Advancement of Science last year and the term "iron hunger" applied to it. Under the supposition that the green coloring matter contains iron, the explanation seems to be simple, but that view cannot be held. Yet the New Zealand investigations seem to show positively that iron is needed. Must we fall back on the *deus ex machina* of modern chemistry and rank the iron as a catalyst? The possible formation of formaldehyde by the action of chlorophyl upon water and carbon dioxide, the ordinary process of plant feeding, so far as the aerial parts are concerned, is discussed in connection with formaldehyde, and the author leans to the view that this is the story of plant nutrition. Incidentally he remarks that it may be possible to form formaldehyde and methanol synthetically upon some similar reaction. This has been completely accomplished as far as methanol is concerned.

The book is too large. A smaller type and structural formulas on much smaller scale would have economized paper and eliminated bulk and weight. The suggested structural formulas of two derivatives of chlorophyl take up over half a page. In a text-book many formulas might be given in a condensed form and the student required to express them graphically. We are told in the preface that the German edition was used by students in advanced chemistry at the University of Edinburgh with such success that it has been translated. This is to be regretted. It should have been kept in the original. Students in any department of advanced chemistry should be obligated to acquire ability to read German and French scientific contributions at sight. The price seems high. Printing is, of course, now costly, but it ought to be possible by economy of space and the use of smaller type to include the same amount of information at a lower figure. The book will be almost exclusively used by young persons whose eyes will not find a type several points lower inconvenient. The translation is excellent.

HENRY LEFFMANN.

CHROMIUM AND ITS CONGENERS. Being Volume 7, part 3, of "A Text-book of Inorganic Chemistry," edited by J. Newton Friend, D.Sc., Ph.D., F.I.C. xxvi–380 pages, 8vo. Philadelphia, J. B. Lippincott Company, 1926. Price, $8.50.

The volume in hand includes chromium, molybdenum, tungsten and uranium, and was prepared by Reece H. Vallance, M.Sc., and Arthur A. Eldridge, B.Sc. The subject-matter follows the forms presented in the numerous volumes already issued. All the topics are treated comprehensively and the literature has been thoroughly sifted. A very useful table, and one which has required much really dreary labor in its compilation, is a comparison of the chronology of the principal periodicals since 1800. This table will be a great aid to research workers who so frequently find imperfect references. All references should give the year as well as volume, if the latter is designated in the publication. Some periodicals give the year only. The system of indicating the several portions of this work by volumes and parts is not commendable. Each issue should be designated as a separate volume. The distribution of the descriptive matter through the several volumes is given in connection with a

presentation of the periodic system as a frontispiece. Text, presswork and paper are all very satisfactory.

It seems to the reviewer that the line has been drawn rather too strictly in the description of the uranium salts. Being a work on "Inorganic Chemistry," it is reasonable that salts of organic acids should not be considered but the double and triple uranium salts containing sodium, having the peculiar property of high insolubility in water, especially in excess of the reagent, seem to require more than the brief notice given. The insolubility of sodium magnesium uranium acetate and its very high molecular weight, of which only a small part is that of the sodium itself, suggest the possibility of direct determination of the latter element. The titration of uranium with permanganate is stated to be much less satisfactory than is expressed in the eleventh edition of Sutton's "Volumetric Analysis" (1924, p. 350).

HENRY LEFFMANN.

CHEMICAL SYNONYMS AND TRADE NAMES. By William Gardner. Third edition, revised and much enlarged. 355 pages, 8vo. New York, D. Van Nostrand Company, 1926. Price, $7.50.

The first edition of this book was issued some years ago. It has been kept up to date by a couple of issues in the intermediate period and now appears in a formal third edition very much enlarged. It is a very useful book. Time was when chemists, pharmacists and manufacturers had in most cases but one name for each of the common chemicals, but the development of chemistry and the necessary extension of nomenclature has brought about a separation of titles. "Butter yellow" is satisfactory to the maker and user; "dimethyl-amino-azo-benzene," the chemist's name, is adapted only to the laboratory.

The book is timely, containing approximately twenty thousand definitions and cross references. The general features of the work were set forth in the review of the first edition.

HENRY LEFFMANN.

PRACTICAL PHYSIOLOGICAL CHEMISTRY: A Book Designed for Use in Courses in Practical Physiological Chemistry in Schools of Medicine and of Science. By Philip B. Hawk, M.S., Ph.D., and Olaf Bergeim, M.S., Ph.D. Ninth edition, revised and enlarged. xviii–931 pages, 8 plates, 273 figures. Philadelphia, P. Blakiston's Son and Company. Copyright, 1926. Price, $6.50.

In the ninth edition of this deservedly popular treatise, Dr. Olaf Bergeim appears as a co-author. Seven new chapters have been added; all the chapters have been revised and enlarged, and several have been entirely rewritten. The new chapters are devoted to: "Physical Chemistry of True and Colloidal Solutions," "Absorption," "Putrefaction and Detoxication," "The Chemistry of the Blood-tissue Analysis," "Respiratory Metabolism and Neutrality Regulation," "The Endocrine Organs," "Energy Metabolism." The chapter on Absorption describes the absorption of the products of digestion from the intestinal tract. The chapter on Putrefaction and Detoxication is devoted to the processes of putrefaction and the mechanism of the animal organism for the conversion of the products of putrefaction into physiologically inert compounds.

The chapter on Respiratory Metabolism and Neutrality Regulation gives a detailed account of the physiological and chemical processes which control the reaction of the blood and maintain its hydrogen-ion concentration and reserve alkalinity. The chapter on Energy Metabolism is devoted to the basal metabolic rate, its significance, and its determination by both open circuit and closed circuit methods. Among the sections which have been added in this edition are those on oxidation and reduction systems, photosynthesis, vitamins and their assay, the biochemical aspects of ultra-violet radiation, and rickets. A concise summary of enzymes, their classification, distribution, and substrates, and the products of their action is given in tabular form. Six vitamins are recognized, including vitamin E (the reproductive or fertility factor) and the pellagra-preventive factor; data concerning the occurrence of the six vitamins in various foods are given in a table. In the description of quantitative methods of analysis, sections are devoted to the principle, the procedure, the calculation and the interpretation of the result. The treatise has an excellent index of twenty-five pages.

Doctor Hawk is now President of the Food Research Laboratories, Inc., New York City, and Doctor Bergeim is Assistant Professor of Physiological Chemistry in the College of Medicine of the University of Illinois at Chicago. In the preparation of this edition, Doctors Hawk and Bergeim have invited eight specialists to write sections of certain chapters, and have thereby increased the value of their book. The treatise is up to date, and of great value, both as a laboratory text and as a reference work, to the physician, the student, and workers in the various divisions of biological chemistry.

JOSEPH S. HEPBURN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report 240, Nomenclature for Aeronautics. By National Advisory Committee for Aeronautics. 77 pages, illustrations, quarto. Washington, Government Printing Office, 1926.

The "Nomenclature for Aeronautics" presented in this report is a revision of the last previous report on the subject (No. 157), which was issued in February, 1923.

This "Nomenclature for Aeronautics" was prepared by a special conference on aeronautical nomenclature authorized by the executive committee of the National Advisory Committee for Aeronautics at a meeting held on August 19, 1924, at which meeting Dr. Joseph S. Ames was appointed chairman of said conference. The conference was composed of representatives of the National Advisory Committee for Aeronautics, and, in response to the committee's invitation, specially appointed representatives officially designated by the Army Air Service, the Bureau of Aeronautics of the Navy Department, the Bureau of Standards, the American Society of Mechanical Engineers, the Society of Automotive Engineers, and the Aeronautical Chamber of Commerce.

This report supersedes all previous publications of the committee on this subject. It is published for the purpose of securing greater uniformity and accuracy in the use of terms relating to aeronautics, in official documents of the government and, as far as possible, in technical and other commercial publications.

Report 243, A Preliminary Study of Fuel Injection and Compression

Ignition as Applied to an Aircraft Engine Cylinder. By Arthur W. Gardiner. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1926. Price, ten cents.

This report summarizes some results obtained with a single cylinder test engine at the Langley Field Laboratory of the committee during a preliminary investigation of the problem of applying fuel injection and compression ignition to aircraft engines. For this work a standard Liberty engine cylinder was fitted with a high compression 11.4:1 compression ratio, piston, and equipped with an airless injection system, including a primary fuel pump, an injection pump, and an automatic injection valve.

The results obtained during this investigation have indicated the possibility of applying airless injection and compression ignition to a cylinder of this size, 5-inch bore by 7-inch stroke, when operating at engine speeds as high as 1850 r.p.m., although the unsuitability of the Liberty cylinder form of combustion chamber for compression ignition research probably accentuated the difficulties to be overcome. No difficulty was experienced in metering and injecting the small quantities of fuel required. A minimum specific fuel consumption with Diesel engine fuel oil of 0.30 pound per I.HP. hour was obtained when developing about 16 B.HP. at 1730 r.p.m. Specific fuel consumption increased for higher loads at these speeds. A maximum power output of 29.7 B.HP. at 1700 r.p.m. was obtained but could not be maintained for more than one-half minute, due to piston failure. Mean effective pressures approaching standard aircraft engine practice could not be obtained, due in part, it was attributed, to the unsuitable form of the Liberty cylinder combustion chamber. Excessive maximum combustion pressures were encountered when developing only about 60 pounds B.M.E.P. at 1700 r.p.m., and piston life was very short. The engine could be idled with regular firing at 400 r.p.m., but acceleration under load was not satisfactory, due probably to the fixed timing of injection during any particular run.

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Chemical Synonyms and Trade Names. A dictionary and commercial handbook by William Gardner, Works Chemist. Third edition, revised and much enlarged. 355 pages, 8vo. New York, D. Van Nostrand Company, 1926. Price, $7.50.

Magnetism and Atomic Structure, by Edmund C. Stoner, Ph.D. 371 pages, illustrations, 8vo. New York, E. P. Dutton and Company, 1926. Price, five dollars.

A Text-book of Organic Chemistry, by Dr. Julius Schmidt, Professor of Chemistry in the Technische Hochschule, Stuttgart. English edition by H. Gordon Rule, Ph.D., D.Sc. 798 pages, 8vo. New York, D. Van Nostrand Company, 1926. Price, nine dollars.

Clinical Application of Sunlight and Artificial Radiation, including their physiological and experimental aspects with special reference to tuberculosis, by Edgar Mayer, M.D. 468 pages, illustrations, plates, 8vo. Baltimore, The Williams and Wilkins Company, 1926. Price, ten dollars.

Stories in Stone Telling of Some of the Wonderlands of Western America and Some of the Curious Incidents in the History of Geology, by Willis T. Lee.

226 pages, illustrations, plates, 8vo. New York, D. Van Nostrand Company, 1926. Price, three dollars.

The New Universe. An outline of the worlds in which we live, by Baker Brownell, Professor of Contemporary Thought, Northwestern University. 455 pages, 8vo. New York, D. Van Nostrand Company, 1926. Price, four dollars.

Practical Structural Design in Timber, Steel and Concrete. A text and reference work for engineers, architects, builders, draftsmen and technical schools; especially adapted to the needs of self-tutored men, by Ernest McCullough, C.E., Ph.D. Third edition, 416 pages, illustrations, 8vo. New York, Scientific Book Corporation, 1926. Price, four dollars.

The Elements of Aerofoil and Airscrew Theory, by H. Glauert, M.A. 228 pages, illustrations, 12mo. Cambridge, University Press, New York, The Macmillan Company, 1926.

A Text-book of Inorganic Chemistry. Edited by J. Newton Friend, D.Sc. Vol. 7, part 3, Chromium and Its Congeners, by Reece H. Vallance and Arthur A. Eldridge. 380 pages, illustrations, 8vo. London, Charles Griffin and Company, Ltd.; Philadelphia, J. B. Lippincott Company, 1926. Price, $8.50.

Elementary Mechanism: A text-book for students of mechanical engineering, by A. T. Woods, M.M.E., and A. W. Stahl, M.E., revised and rewritten by Philip K. Slaymaker, M.E. 250 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1926. Price, three dollars.

Physico-chemical Methods, by Joseph Reilly, D.Sc., William Norman Rae, M.A., and Thomas Sherlock Wheeler, Ph.D. 735 pages, illustrations, 8vo. New York, D. Van Nostrand Company, no date. Price, eight dollars.

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Les Ions d'Hydrogène. Signification — Mesure — Applications — Données Numeriques, par W. Kopaczewski, Professeur à l'Institut des Hautes Études de Belgique. 322 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1926. Price, forty francs.

Canada, Department of Mines, Mines Branch. Sodium Sulphate of Western Canada, Occurrence, Uses, and Technology, by L. Heber Cole. 160 pages, illustrations, plates, maps, 8vo. Ottawa, King's Printer, 1926. Price, forty cents.

U. S. Secretary of Agriculture. Report for 1926. 120 pages, 8vo. Washington, Government Printing Office, 1926.

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U. S. Department of Commerce. A Standard State Zoning Enabling Act under Which Municipalities May Adopt Zoning Regulations by the Advisory Committee on Zoning. [Revised edition, 1926.] 13 pages, 8vo. Washington, Government Printing Office, 1926. Price, five cents.

U. S. Department of Commerce, Bureau of Standards. Circular No. 103, United States Government Master Specification for Varnish, Spar, Water

Resisting. 6 pages, 8vo. Washington, Government Printing Office, 1926. Price, five cents.

National Advisory Committee for Aeronautics. Technical Notes No. 249, Effect of Protruding Gasoline Tanks upon the Characteristics of an Airfoil, by Eastman N. Jacobs. 2 pages, illustrations, quarto. No. 250, Influence of the Orifice on Measured Pressures, by Paul E. Hemke. 7 pages, illustrations, photographs, quarto. No. 251, The Effect of Tube Length upon the Recorded Pressures from a Pair of Static Orifices in a Wing Panel, by T. Carroll and R. E. Mixon. 4 pages, photograph, quarto. Washington, Committee, 1926.

Food Research Institute, Stanford University. A General Statement. 12 pages, 8vo. Stanford University, Calif., 1926.

A Trichromatic Colorimeter Suitable for Standardization Work, by J. Guild, A.R.C.Sc. 24 pages, 8vo. Reprinted from the *Transactions of the Optical Society,* vol. 27, No. 2, 1925–1926. Cambridge, University Press.

The Metric Advance. The move to standardize and decimalize the yard-quart-pound on the world metric basis. Testimony presented to the Congress of the United States of America by Aubrey Drury, Director, All-America Standards Council. 22 pages, 8vo. San Francisco, 1926.

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